Predictability and Science

- The Universe *is* predictable
- Prediction is a major goal of science
- Model building

What is a model?

- Mathematical equations
- Computer programs

Why create a model of some phenomenon?

- So we can understand the phenomenon better
- So we can predict the future

How can we tell if we have a good model?

- Compare the model's predictions with experimental outcomes

How to decide between competing models?

- Easy if one fits the data better than others
- Harder if all models fit the data equally well
- Gather more data
- Devise experiments to distinguish between models
  (one model predicts outcome A, other predicts outcome B)
- Occam's Razor

Science in the 19th century

- Newton's laws of motion
- The “clockwork universe”

An intellect knowing all the forces acting in nature at a given instant, as well as the momentary positions of all things in the universe, if this intellect were vast enough to submit these data to analysis, would be able to comprehend in one single formula the motions of the largest bodies as well as the lightest atoms in the world; to it nothing would be uncertain, and the future as well as the past would be present before its eyes.

—*Pierre-Simon Laplace*

- Late-19th-century physicists thought they were basically “done”

New developments in the 20th century

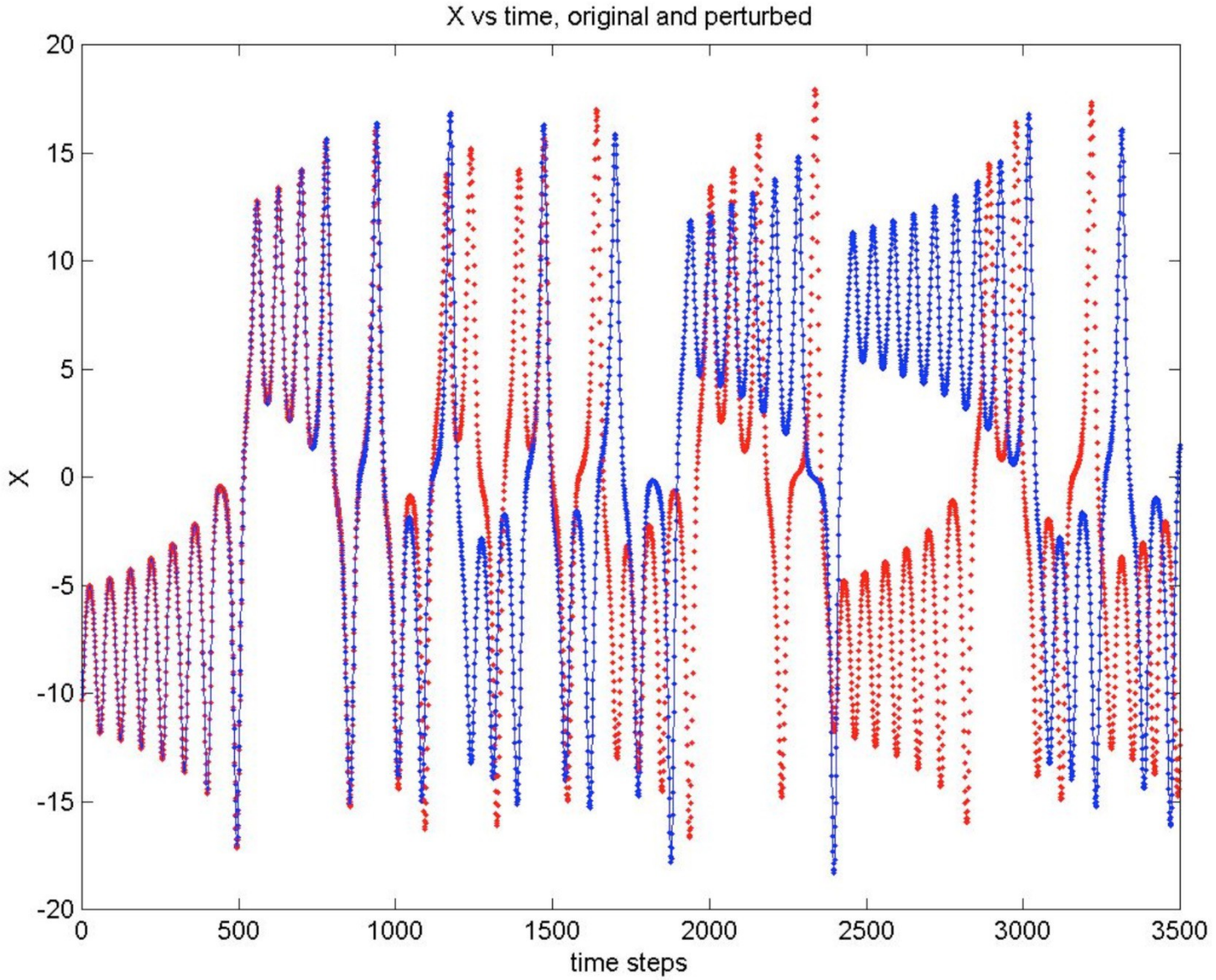
- Relativity (Einstein)
- Quantum mechanics (Planck, Einstein, Bohr, Heisenberg, others)
- Heisenberg's Uncertainty Principle (1927)
- Gödel's incompleteness theorem (1931)
- Turing's halting problem (1936)

Chaos theory (1970s)

- Sensitive dependence on initial conditions
- Stable and unstable systems
- 2-body problem (https://evgenii.com/blog/two-body-problem-simulator)
- 3-body problem (https://evgenii.com/blog/three-body-problem-simulator)

Edward Lorenz's 1963 computer model of the weather

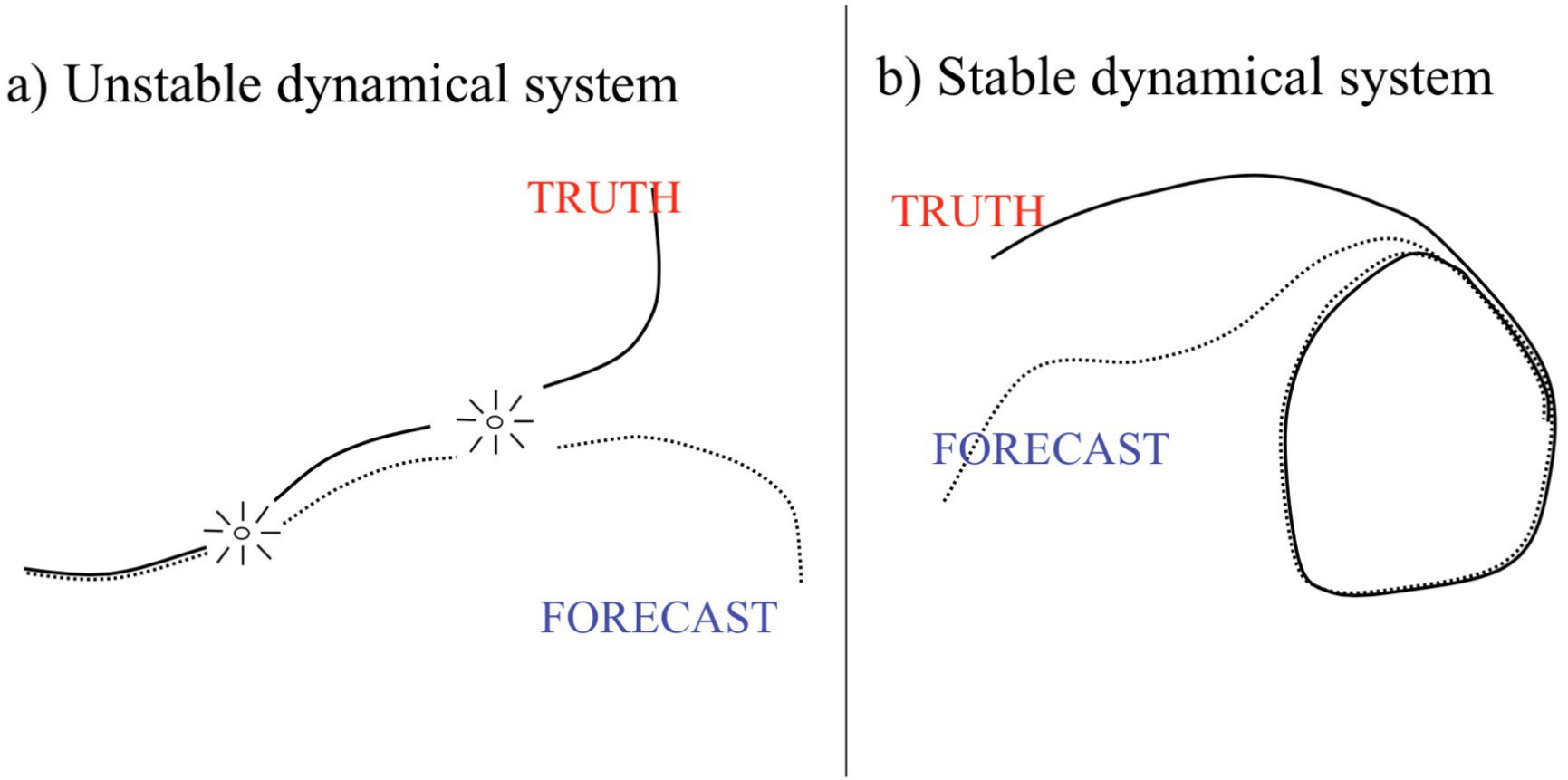
- Simple mathematical model with three variables *x, y,* and *z*



https://www.atmos.umd.edu/~ekalnay/pubs/Chaos-Predictability-EnKF-WMOtalk.pdf

Central theorem of chaos (Lorenz, 1960s)

- Unstable systems have finite predictability (chaos)
- Stable systems are infinitely predictable



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Definition of deterministic chaos (Lorenz)

When the present determines the future

BUT

the approximate present does not approximately determine the future

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Principle features of chaotic systems

- Nonlinearity
- Feedback

- **Nonlinearity: a silly example**
- “Linear brick pile”
- We can understand the effect of a new brick on the whole pile, without knowing how many other bricks are already there
- “Nonlinear brick pile”
- We cannot understand the effect of adding a new brick without knowing about the other bricks that are already there

Nonlinearity: a more realistic example

- Logistic model from population biology
- n_t = population size at time t
- k = carrying capacity of the environment

- Example: birth rate 2, death rate 0.4, carrying capacity 32

$$n_{t+1} = (R_{birth} - R_{death}) \frac{k\, n_t - {n_t}^2}{k}$$

The Logistic Map

- Just the logistic model in a simpler form
- $x_t = n_t / k$ = fraction of carrying capacity at time t
- x always stays somewhere in the range from 0 to 1
- $R = R_{birth} - R_{death}$

$$x_{t+1} = R\, x_t (1 - x_t)$$