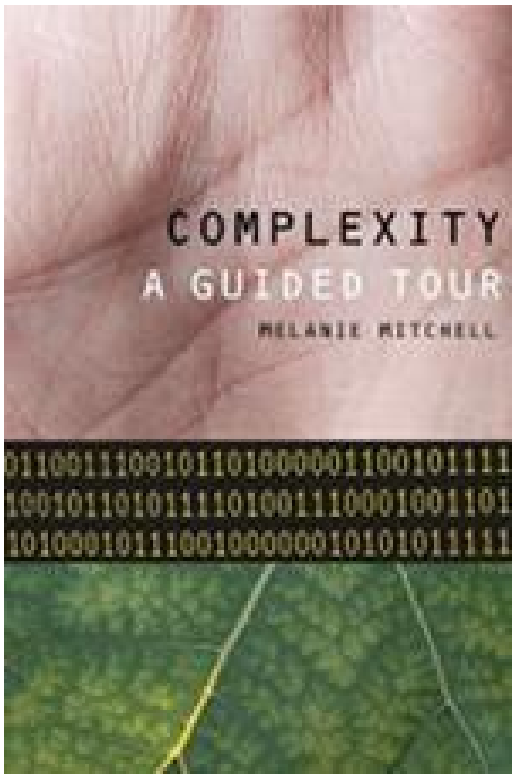
This Week's Reading

- Chapters 3-4 of *Complexity: A Guided Tour*



Computation: The Big Questions

- What is the relationship between information processing in computers and in natural systems?
- What does “computation” mean, exactly?
- What are the limits of computation, if any? Are there things that cannot be computed, even in principle?

The Big Questions in 1900

- Is mathematics **complete**?

  Can every mathematical statement be proved or disproved from a finite set of axioms and rules?

- Is mathematics **consistent**?

  Can only *true* statements be proved?

- Is mathematics **decidable**?

David Hilbert (1862-1943)

Is there a *definite procedure* that can be applied to every statement that will tell us in a finite time whether the statement is true or false?

The *Entscheidungsproblem*

- Is there a *definite procedure* that can be applied to every statement that will tell us in a finite time whether the statement is true or false?

- What is a “definite procedure”?
  - A set of precise instructions for accomplishing some task
  - A recipe
  - An algorithm

The *Entscheidungsproblem*

- Is there a *definite procedure* that can be applied to every statement that will tell us in a finite time whether the statement is true or false?

- What is a "definite procedure"?
  - A set of precise instructions for accomplishing some task
  - A recipe
  - An algorithm
  - A **Turing machine**

- Alan Turing discovered that for many well-defined problems, no definite procedure exists

Alan Turing (1912-54)

Alan Turing

- Published *"On Computable Numbers, with an Application to the Entscheidungsproblem"* in 1936
  - This paper founded the field of computer science
  - Introduced the concept of a Turing machine
  - Provided a blueprint for building real electronic digital computers
- Worked secretly for the British government on deciphering German military codes during WWII
  - Succeeded in breaking German encryption schemes
  - Allies could read German communications from 1942 on
  - May well have turned the tide of the war

Alan Turing

- Published *"Computing Machinery and Intelligence"* in 1950
  - Founded the field of Artificial Intelligence
  - Introduced the Imitation Game (a.k.a. the Turing Test)
  - Clearly foresaw many later developments in AI
- Did groundbreaking work on self-organizing chemical reactions in the early 1950s
- Prosecuted by the British government in 1952 for being homosexual
  - Forced to undergo hormone "therapy"
  - Committed suicide in 1954 at the age of 41
  - British government formally apologized on September 10, 2009

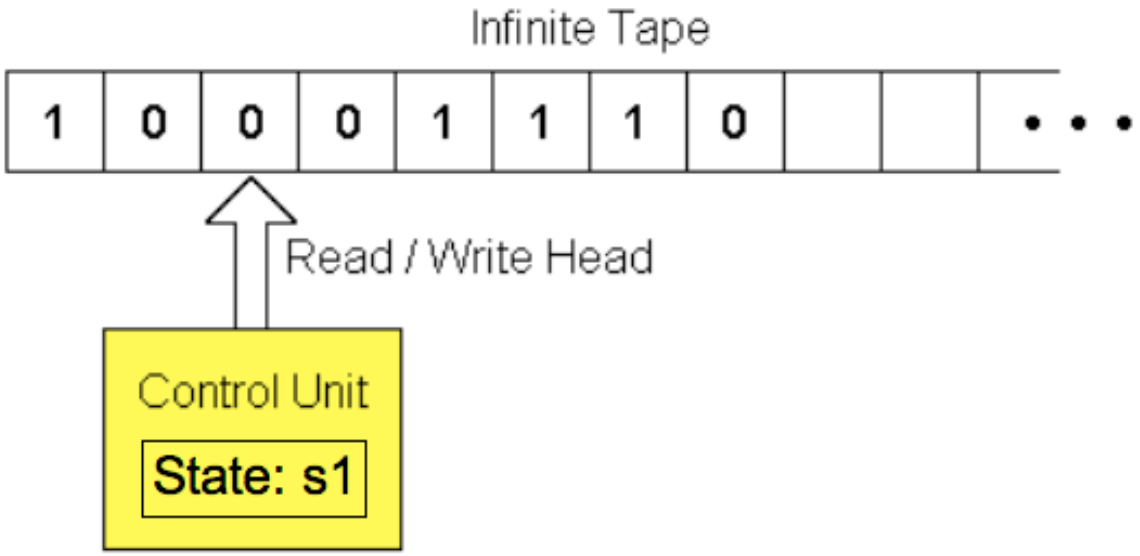
Alan Turing

- *Thousands of people have come together to demand justice for Alan Turing and recognition of the appalling way he was treated. While Turing was dealt with under the law of the time and we can't put the clock back, his treatment was of course utterly unfair and I am pleased to have the chance to say how deeply sorry I and we all are for what happened to him ... So on behalf of the British government, and all those who live freely thanks to Alan's work I am very proud to say: we're sorry, you deserved so much better.*

  —British Prime Minister Gordon Brown
  September 10, 2009

- Queen Elizabeth II granted Turing an official royal pardon on December 24, 2013

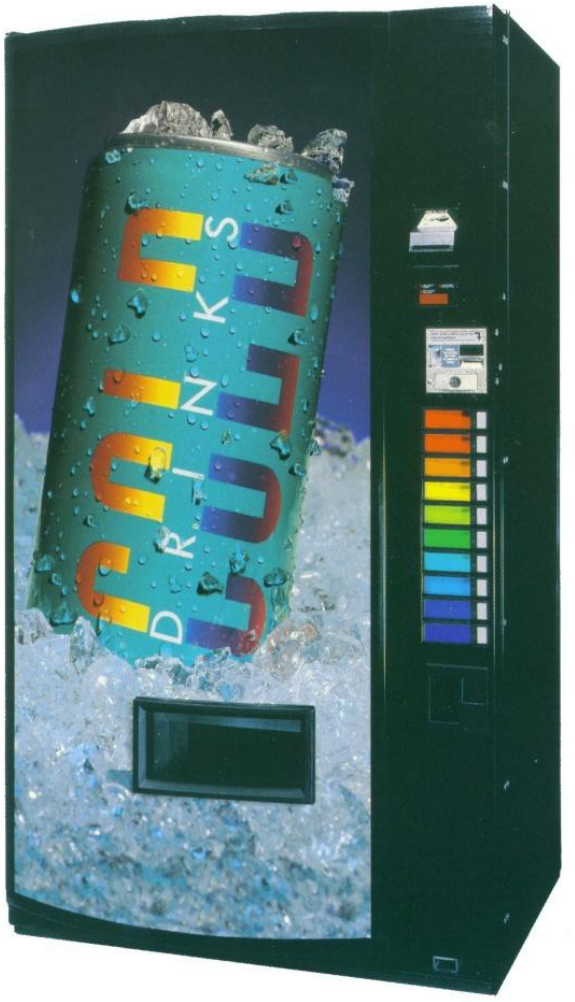
Turing Machines



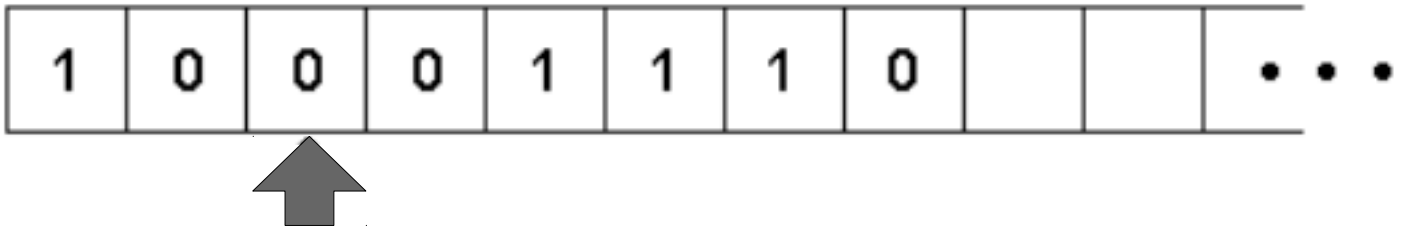
- A Turing machine consists of:
  - A finite set of **states** that it can be in
  - A finite set of **symbols** that it can read and write
  - An infinite **tape** divided into cells for storing symbols
  - A finite **rule table** that tells the machine what to do

States

- How does a **vending machine** "know" whether you have put in the correct amount of change?
- Sometimes when you press a button, nothing happens
- Other times when you press the same button, the machine gives you a soda
- The machine can be in one of several possible internal **states:**
  - "Insufficient money received"
  - "Exact amount received"
  - "Too much money received"



Symbols and the Tape



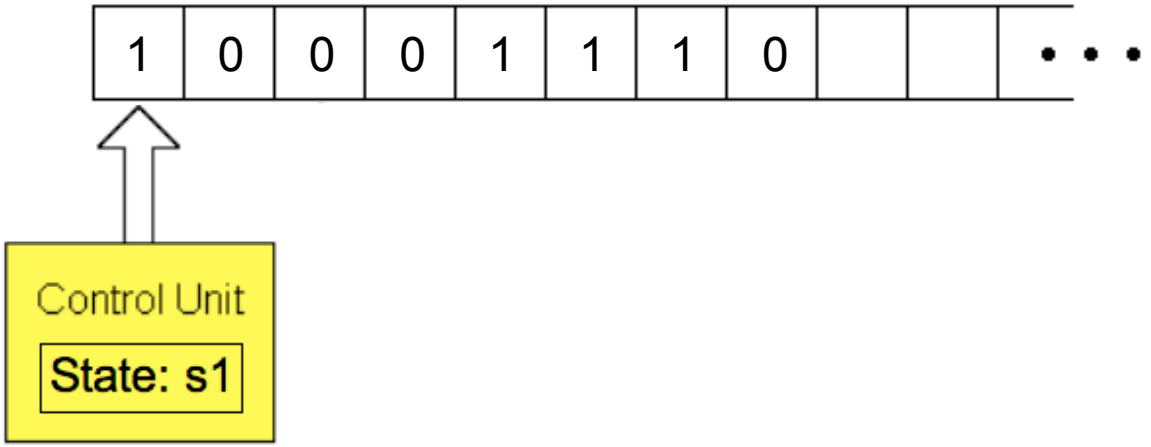
- Symbols are usually just **0**, **1**, and **blank**
- Any arbitrary set of symbols can be used

  … as long as the set is **finite**
- The tape stores **one symbol per cell**

  … but has an **infinite number of cells** available
- The tape head **reads or writes** one symbol at a time

  … and then moves **left, right,** or makes **no move**

An Example



States: **s1**, **halt**

Rules:

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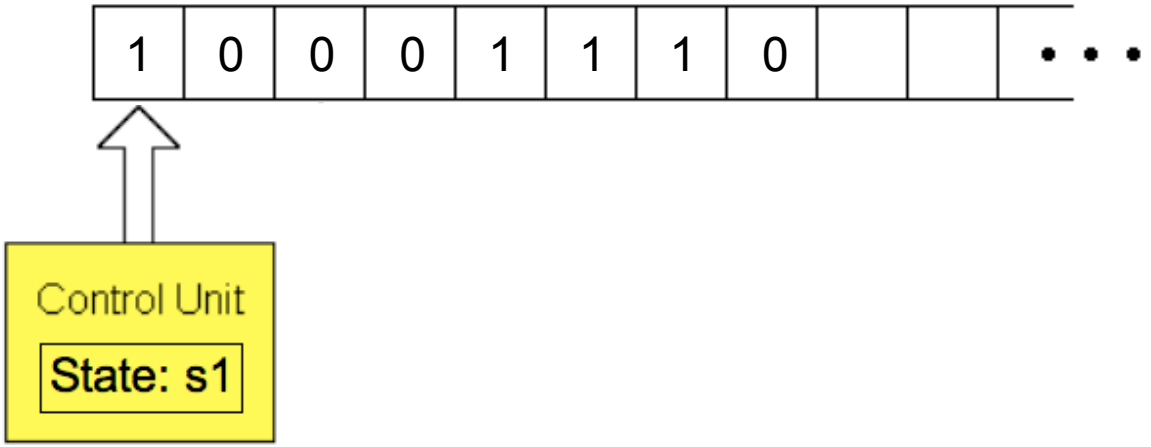

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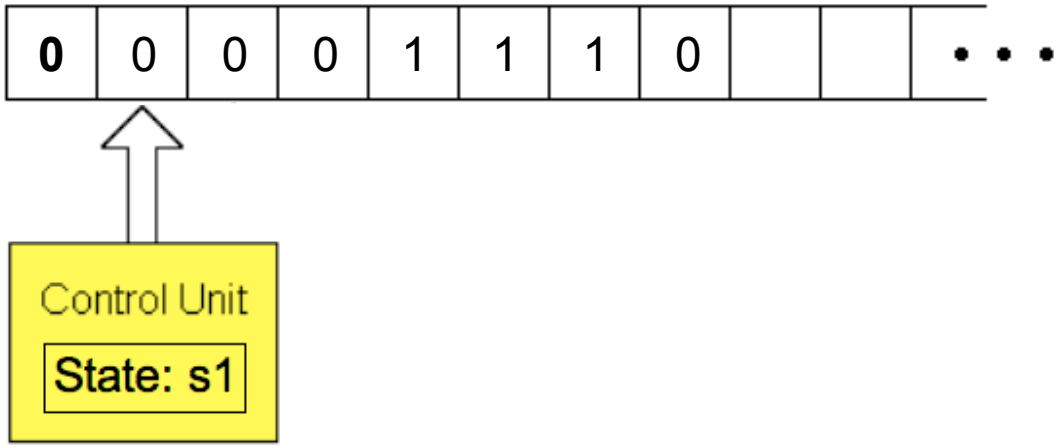

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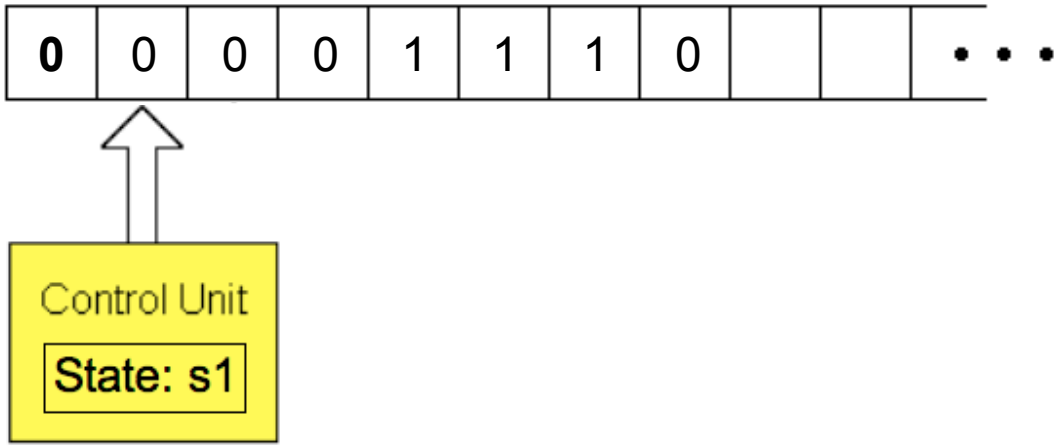

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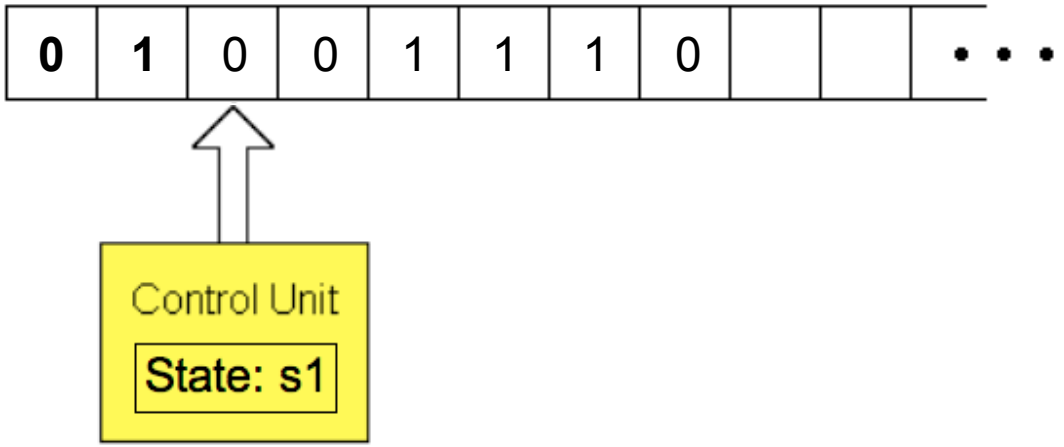

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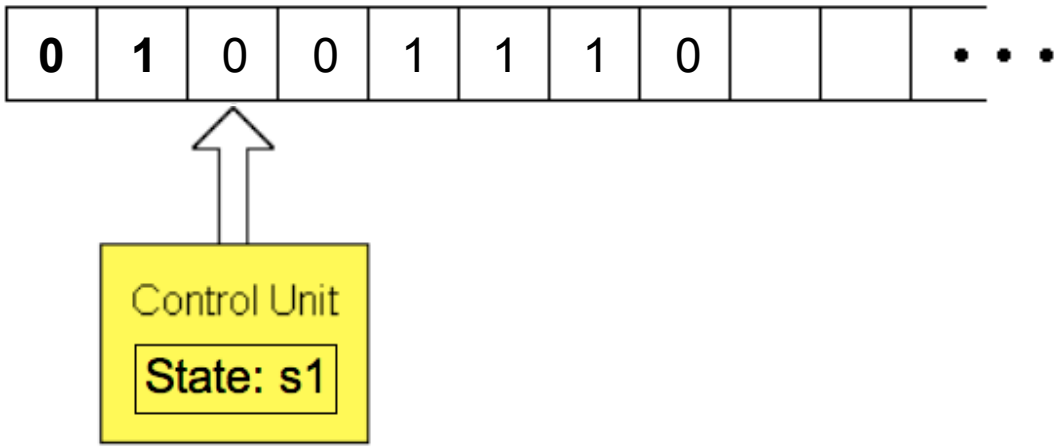

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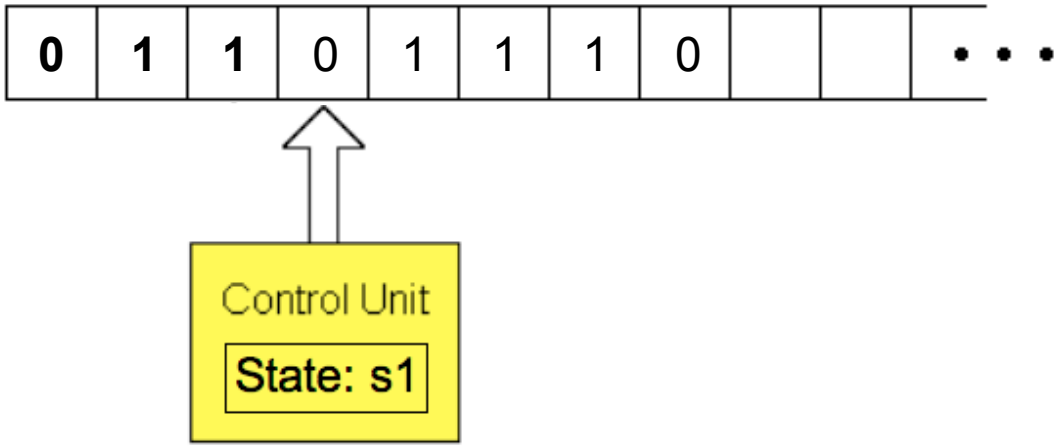

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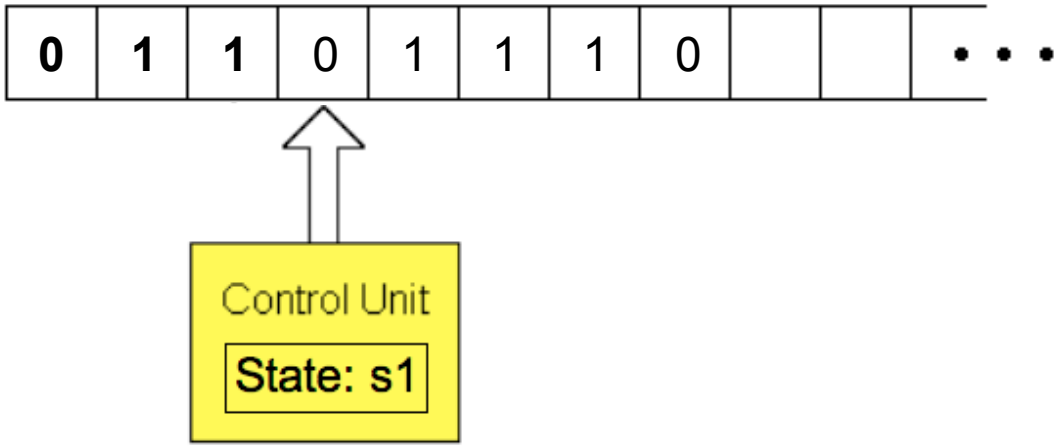

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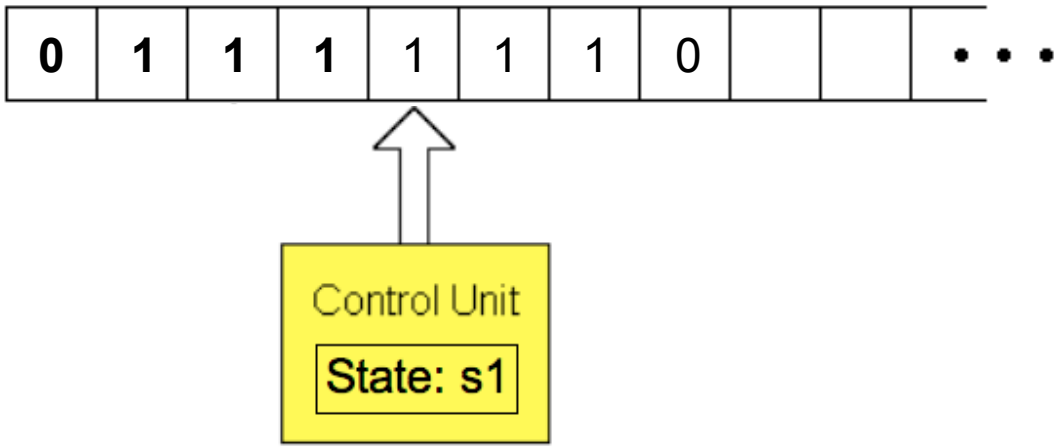

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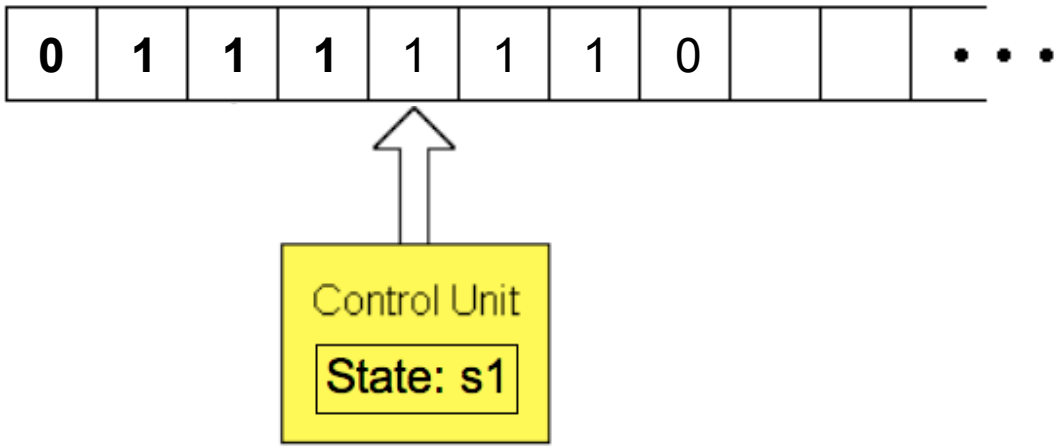

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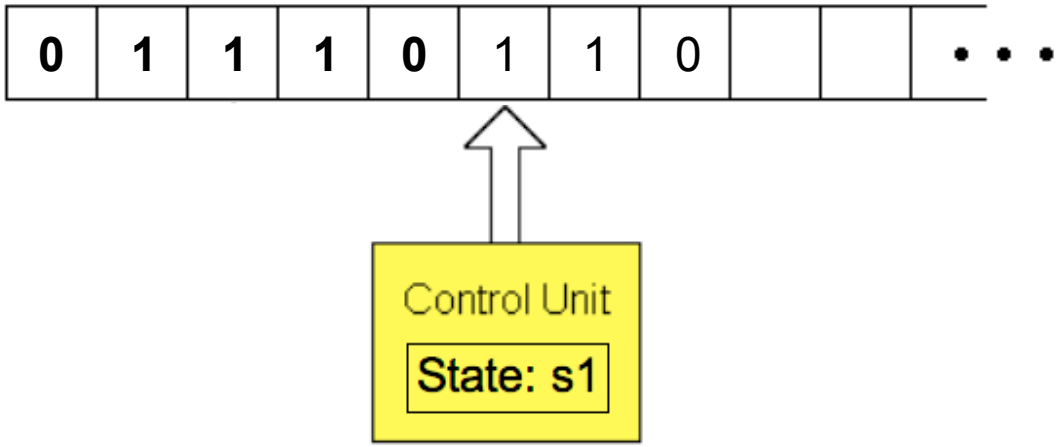

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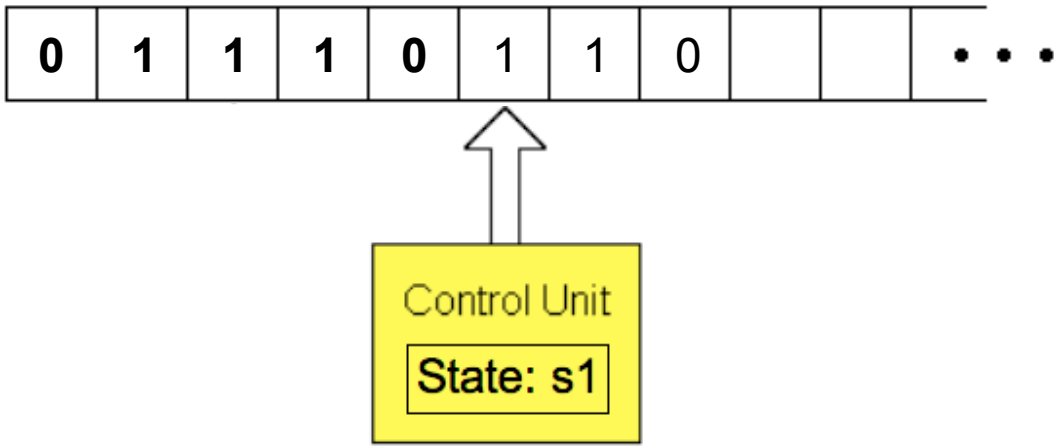

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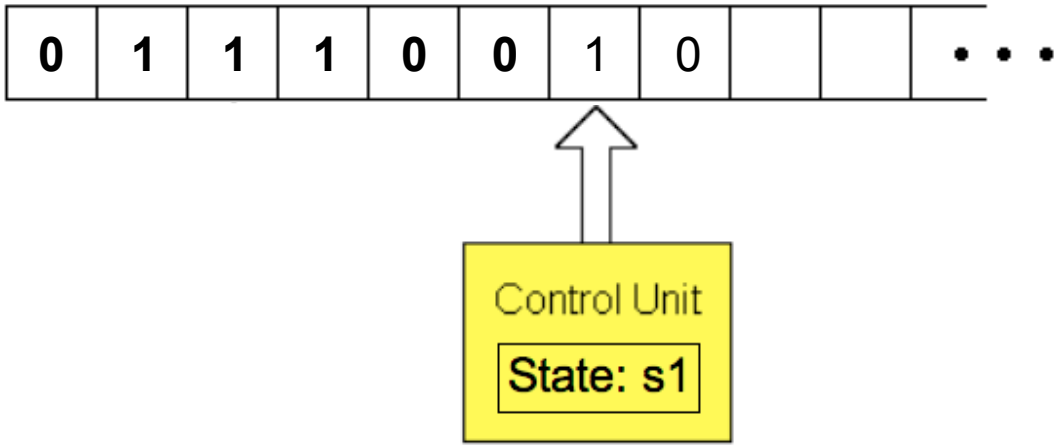

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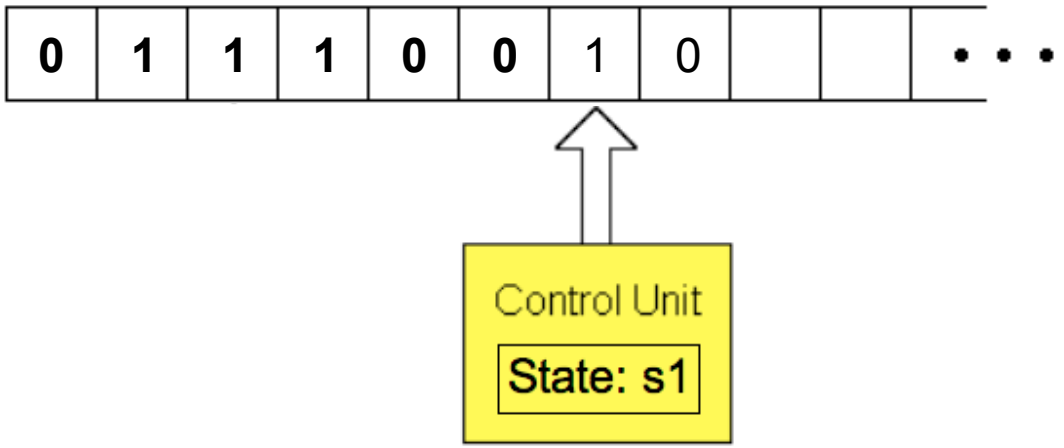

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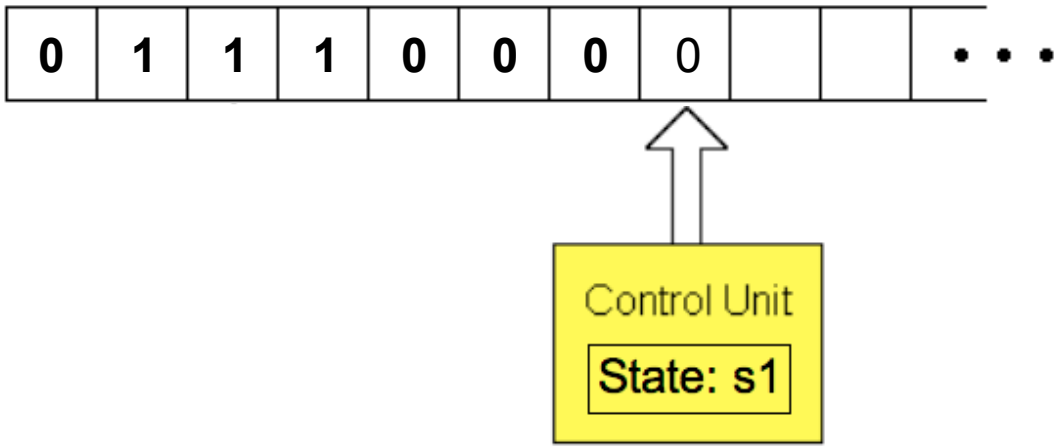

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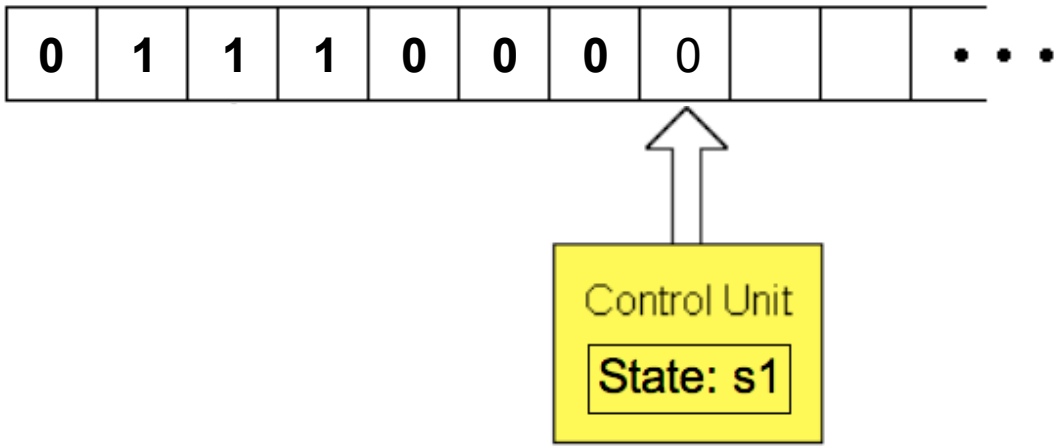

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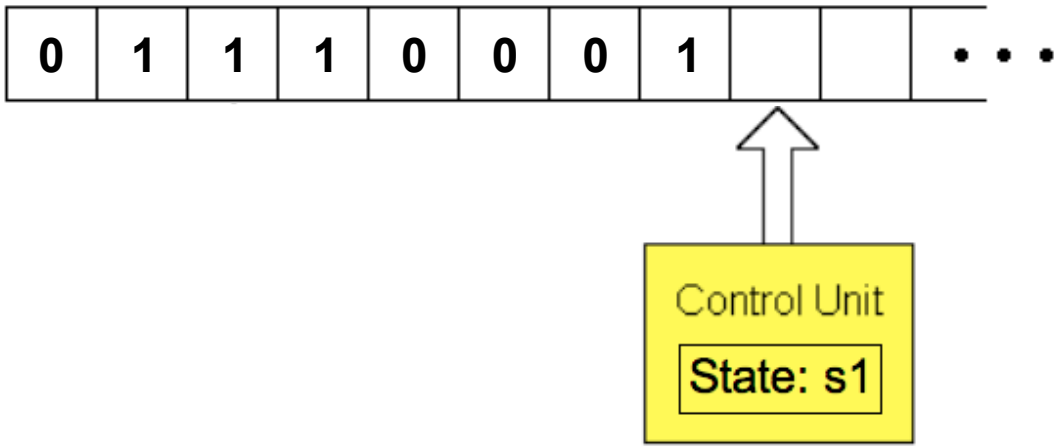

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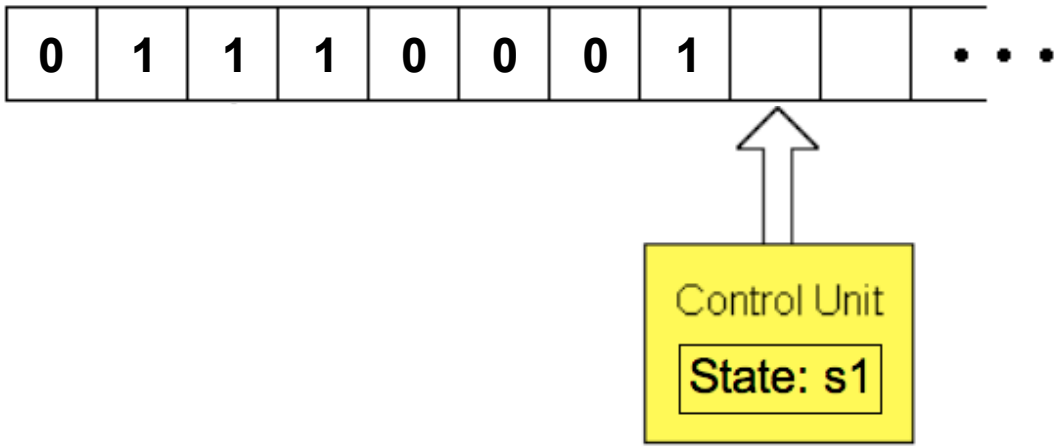

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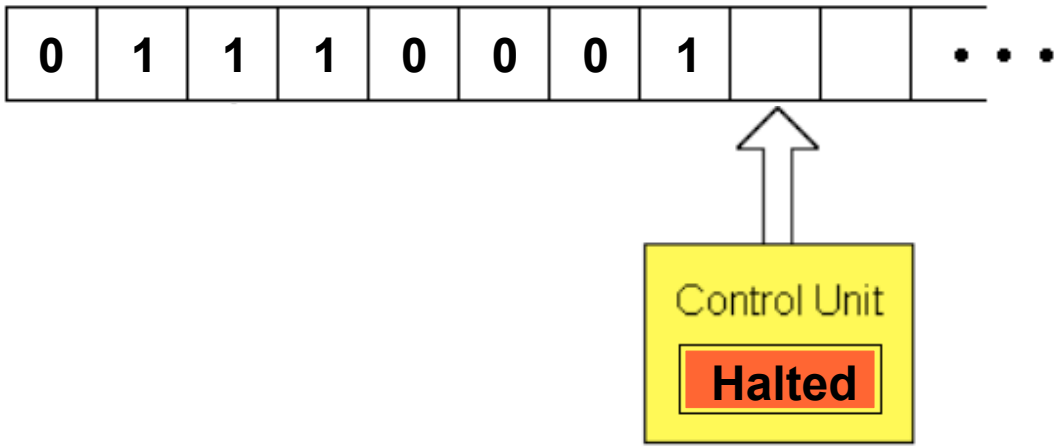

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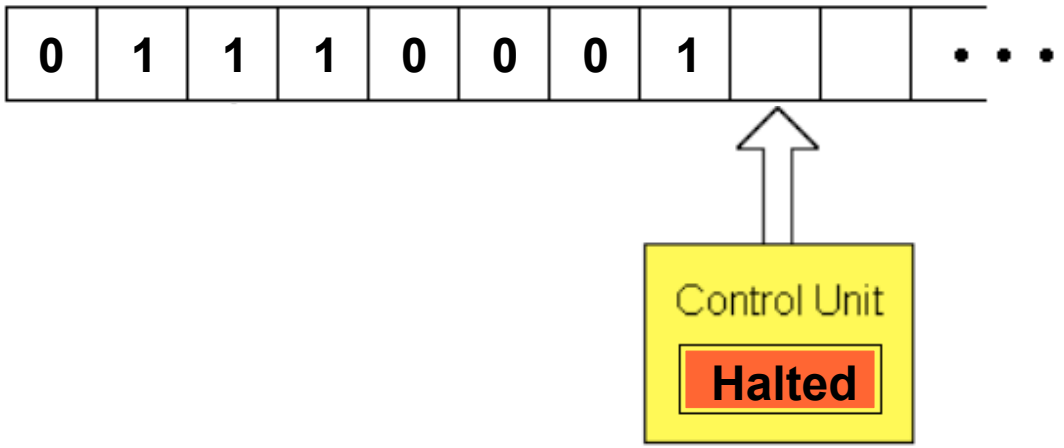


This machine performs the “binary inversion” function

10001110 → 01110001

000 → 111

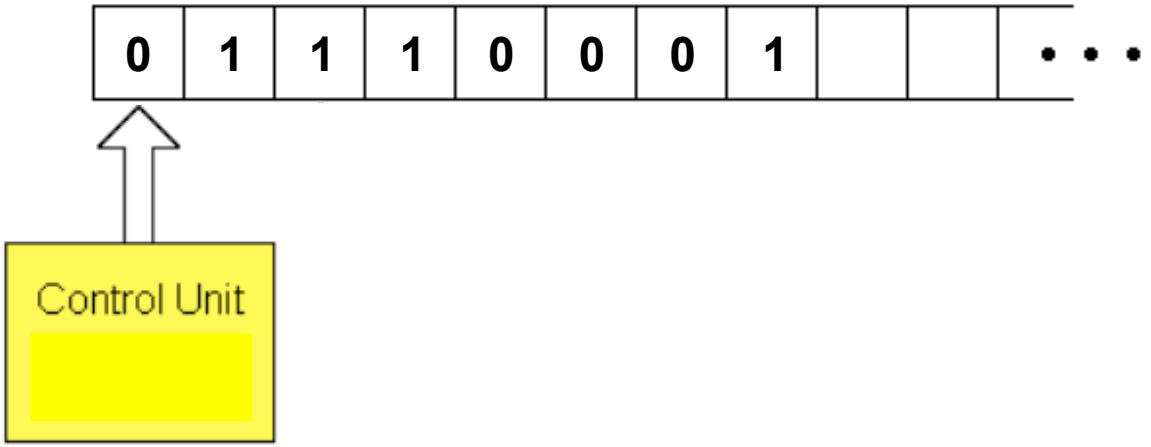
1100 → 0011

1 → 0

0101010 → 1010101

etc...

Your Turn: Design an “Eraser” Machine



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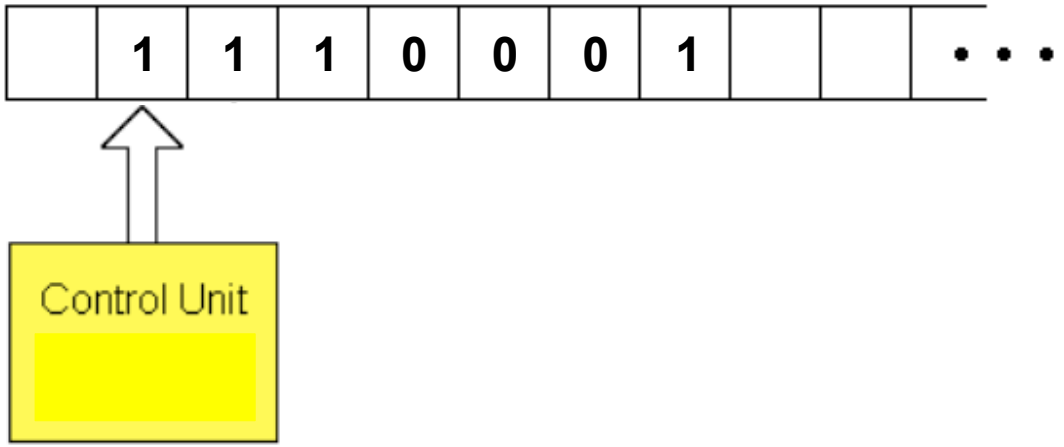

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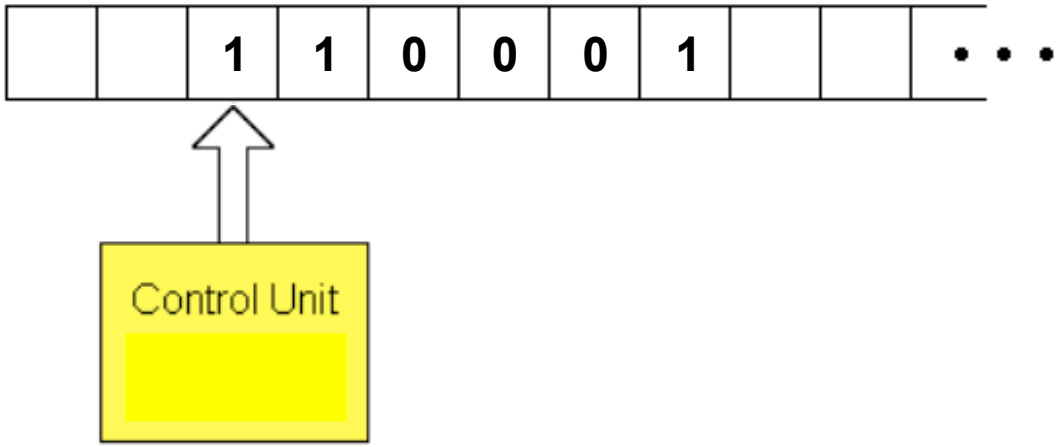

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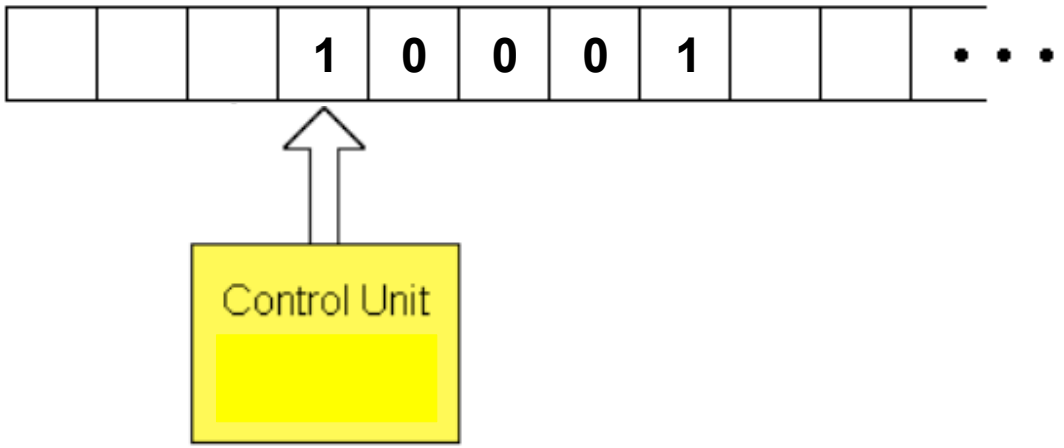

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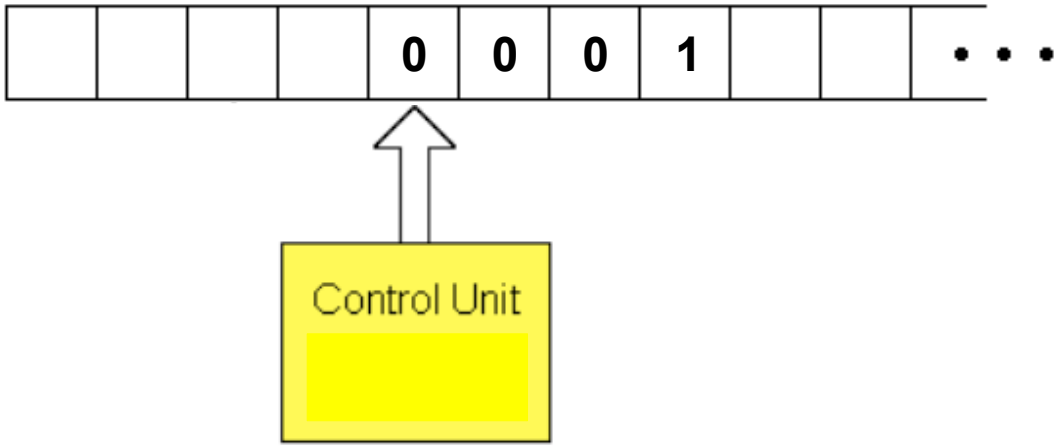

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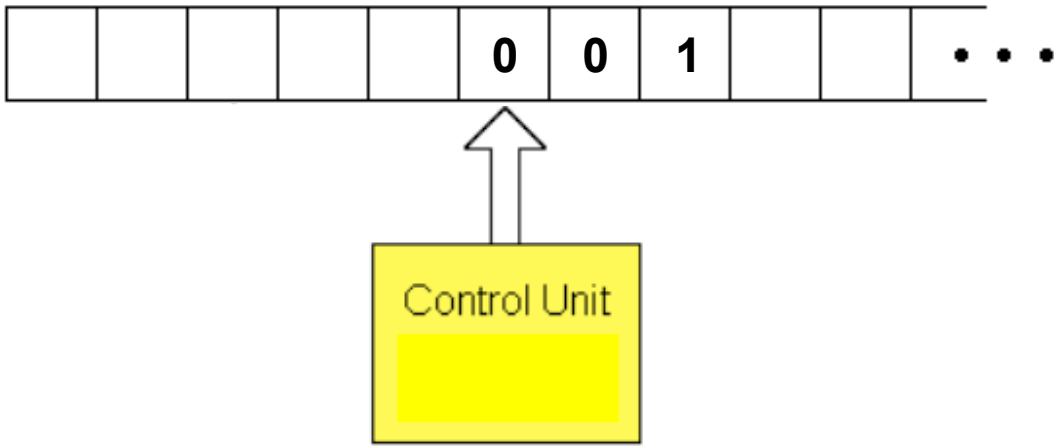

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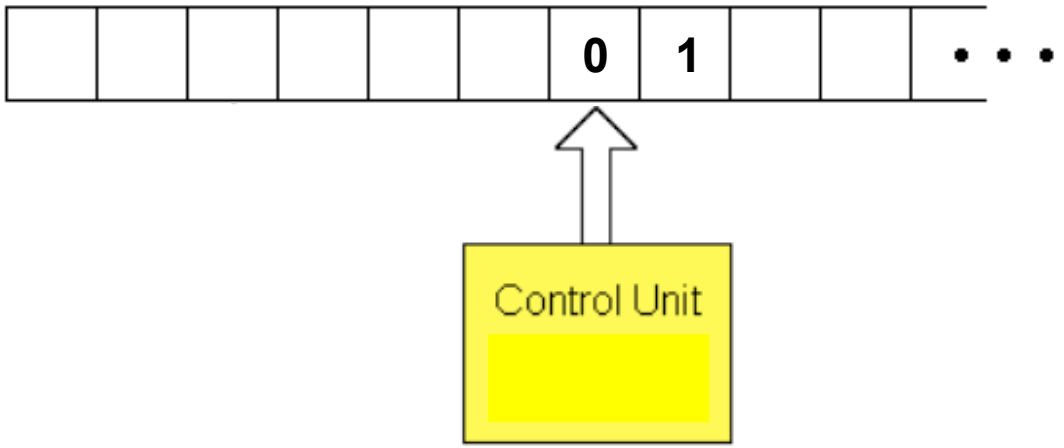

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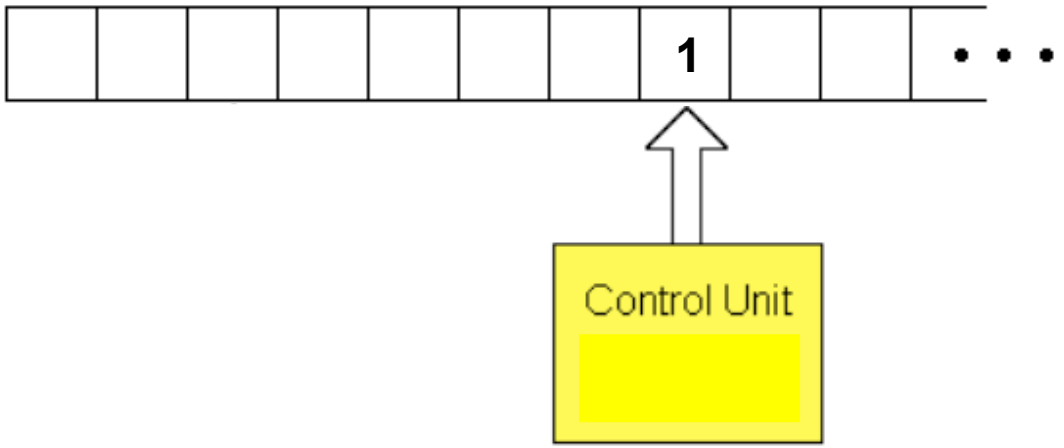

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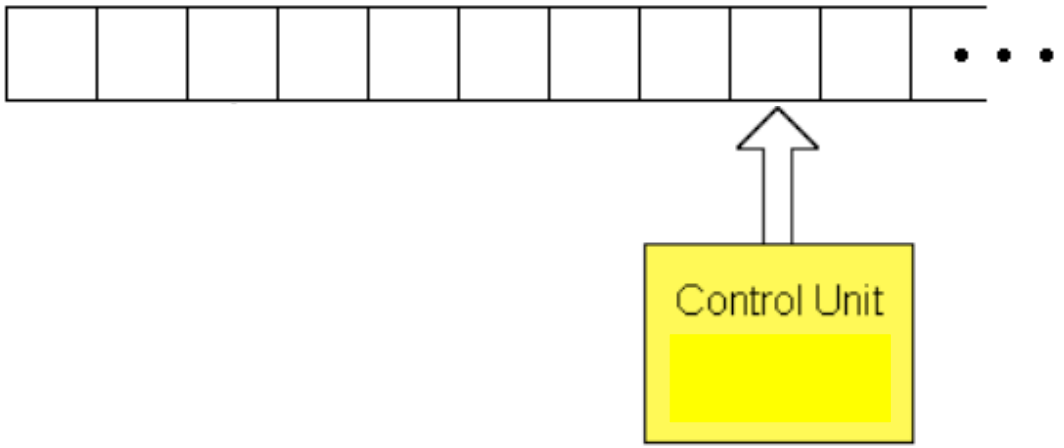

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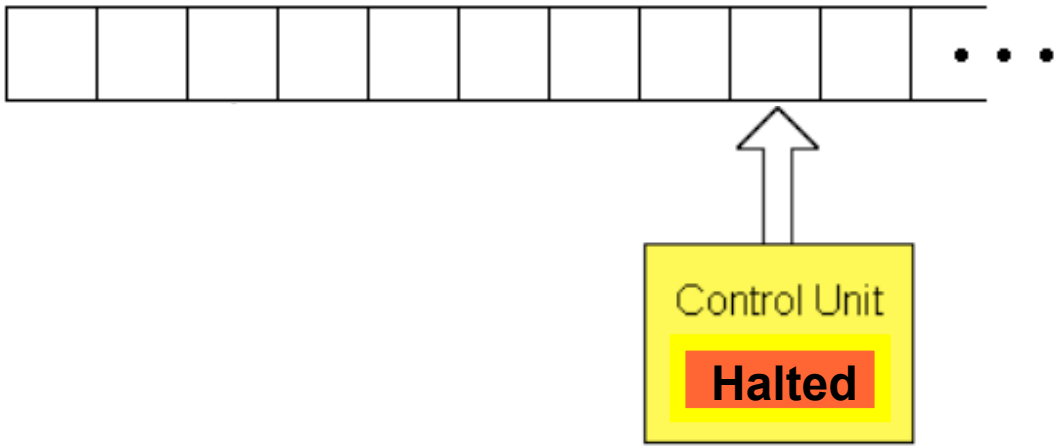

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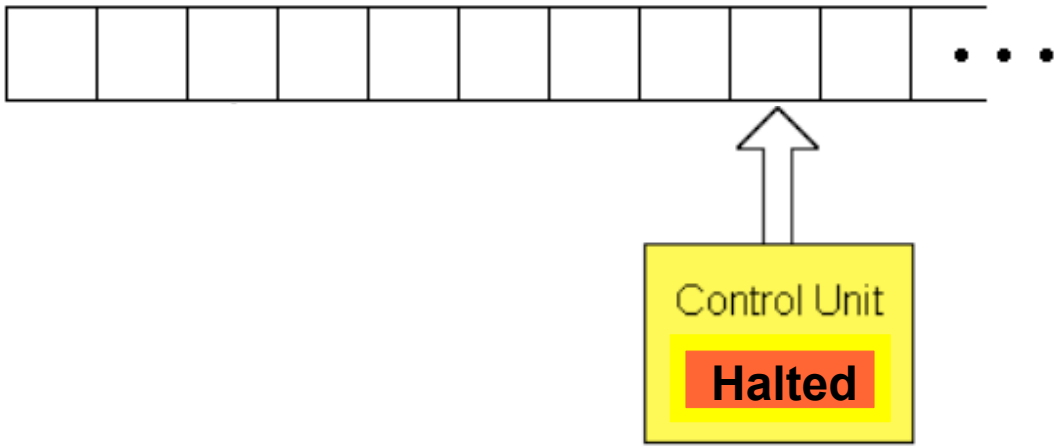

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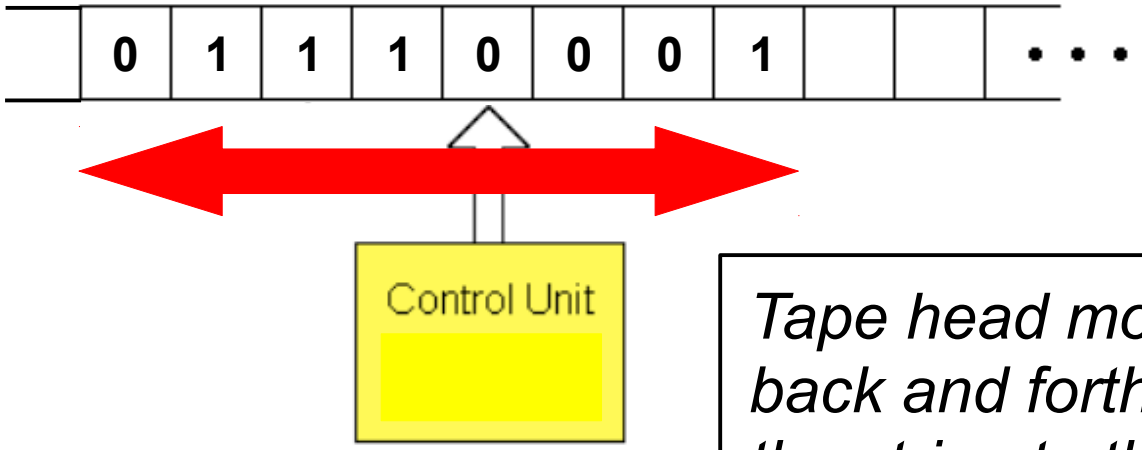
s1	**1**	***blank***	**right**	**s1**
s1	***blank***	***blank***	**none**	**halt**

How About a “Zigzag” Machine?

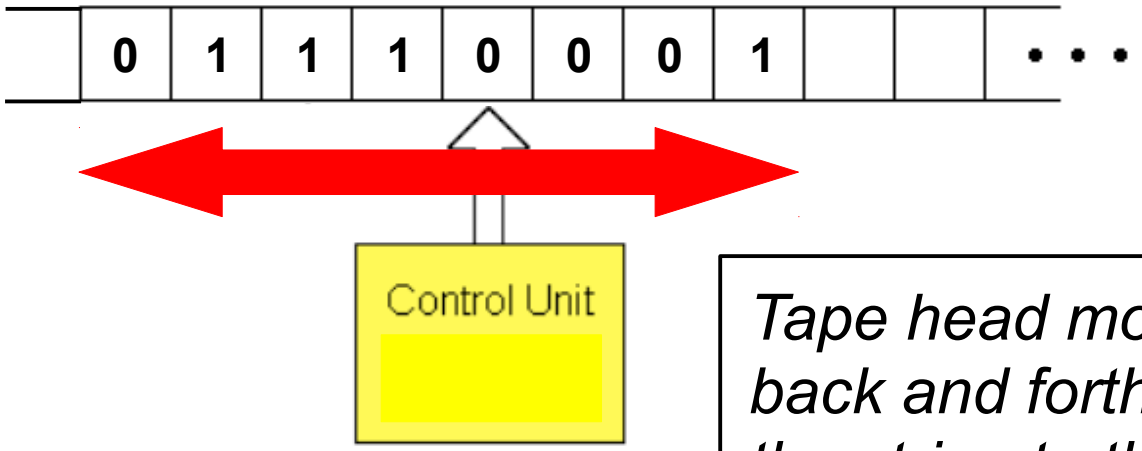


Tape head moves endlessly back and forth from one end of the string to the other, without changing any symbols

States:

current state *current symbol* *new symbol* *move* *new state*

How About a “Zigzag” Machine?

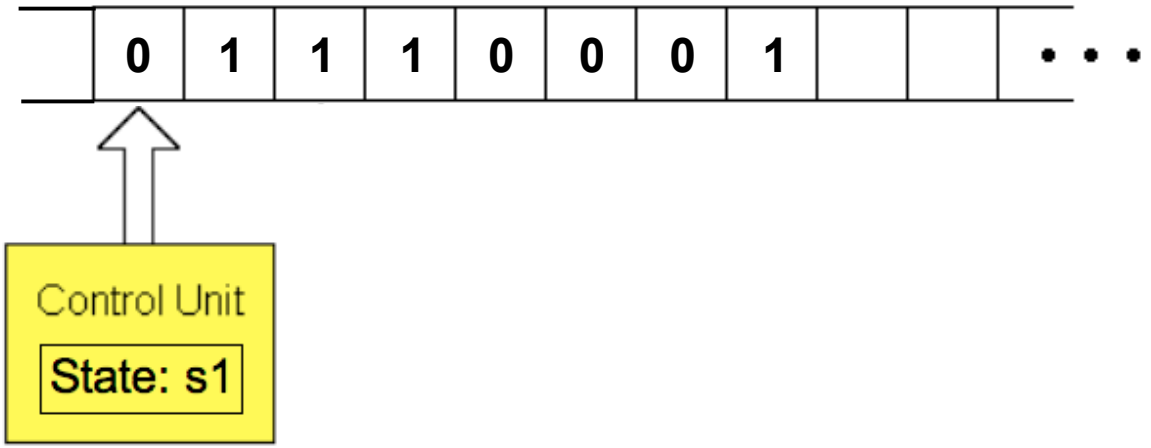


Tape head moves endlessly back and forth from one end of the string to the other, without changing any symbols

States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

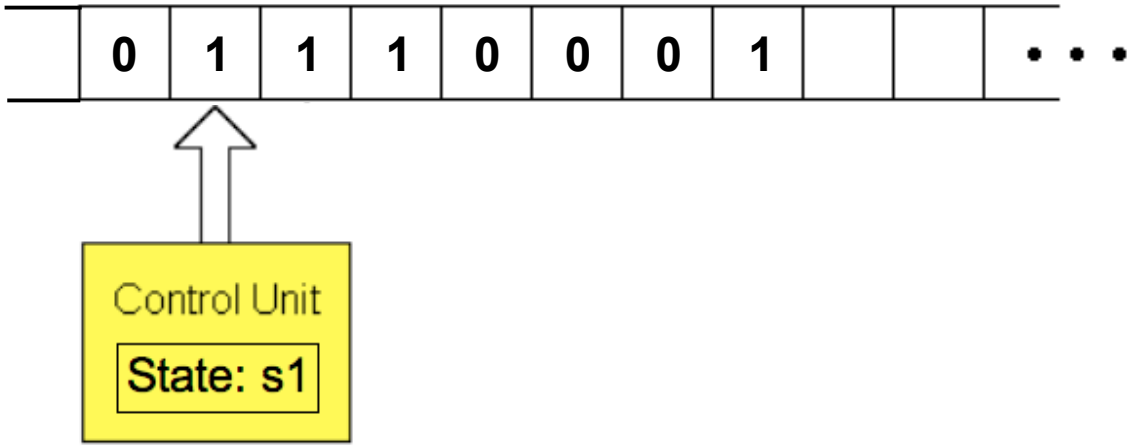
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

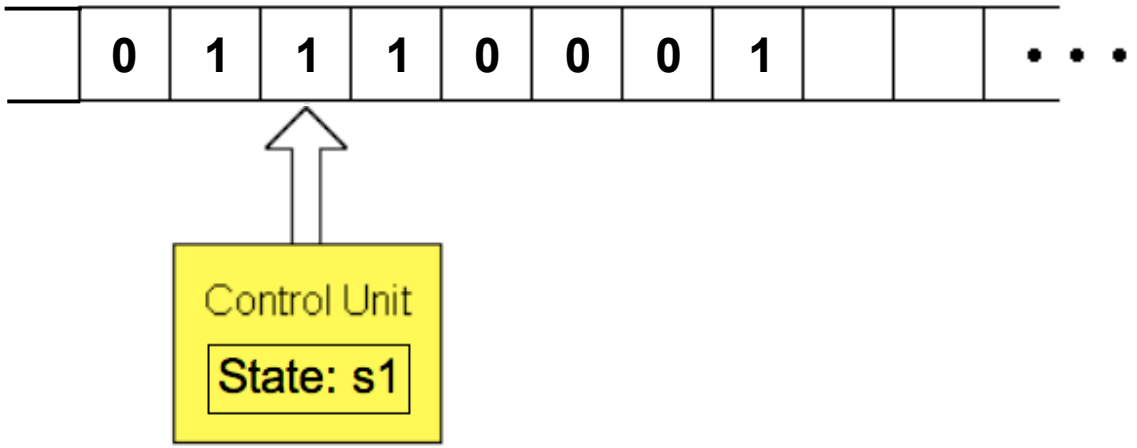
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

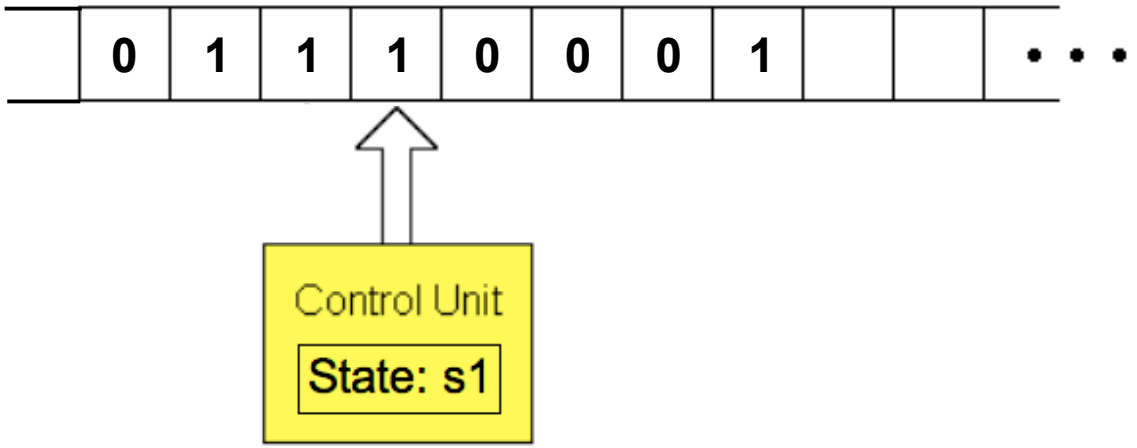
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

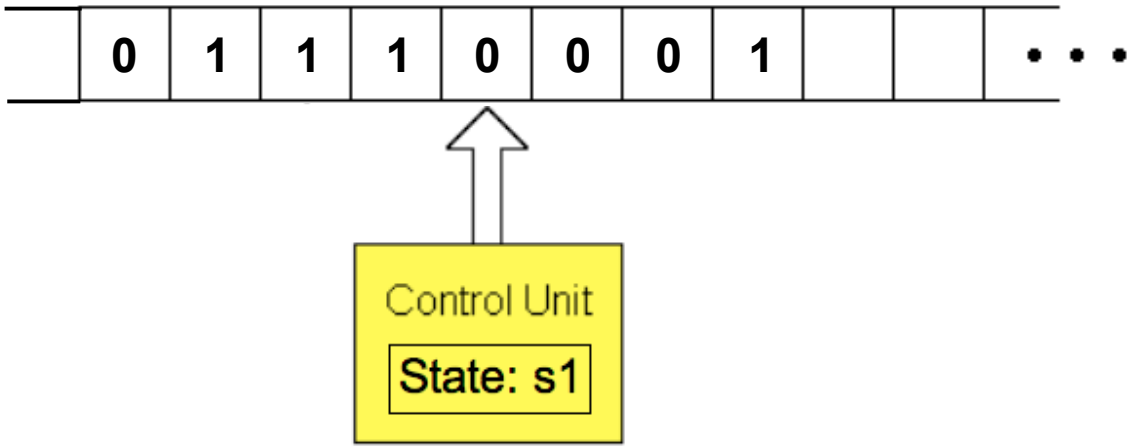
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

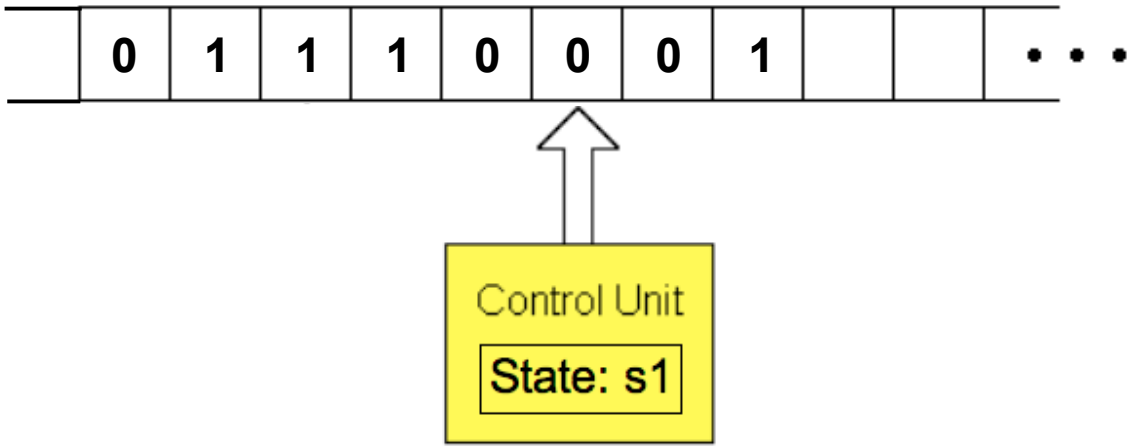
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

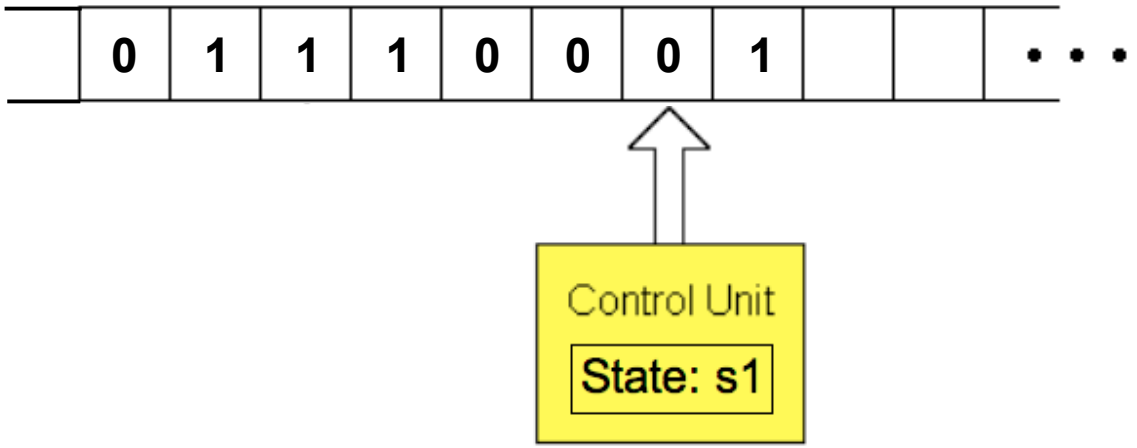
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

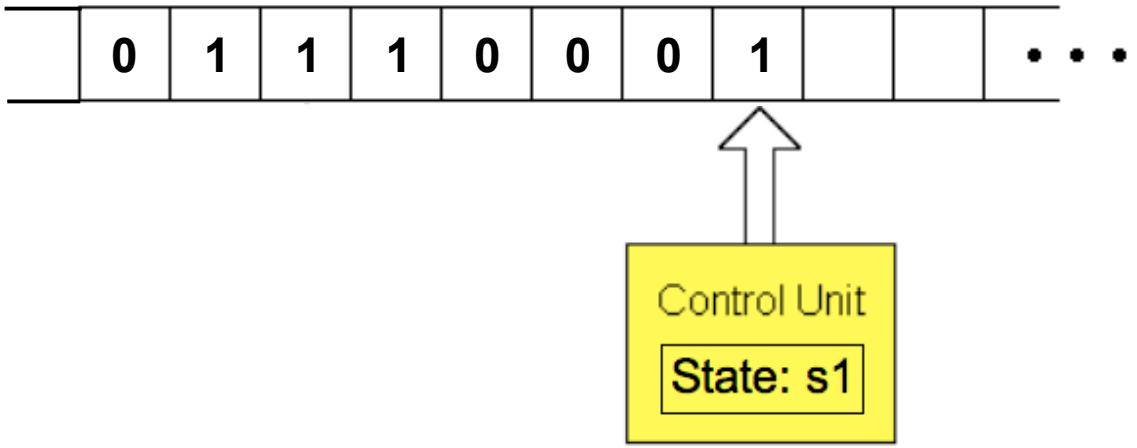
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

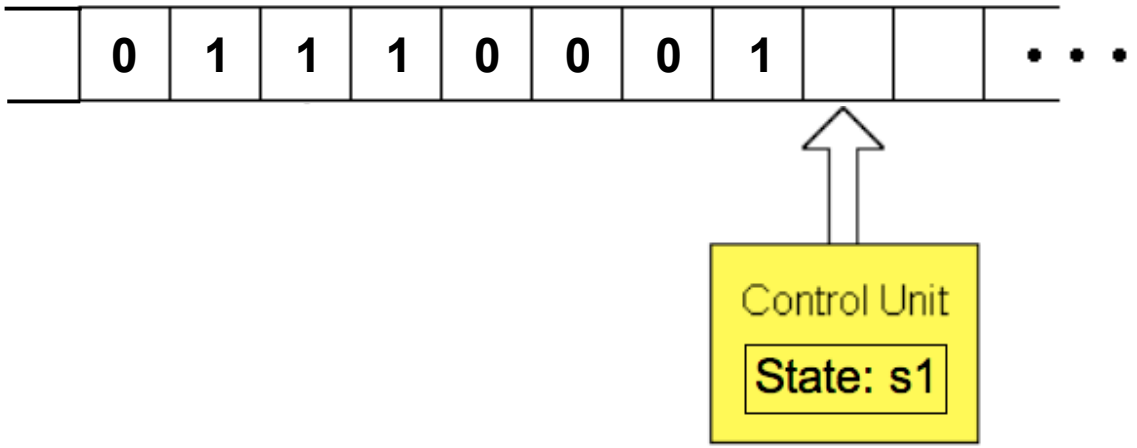
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

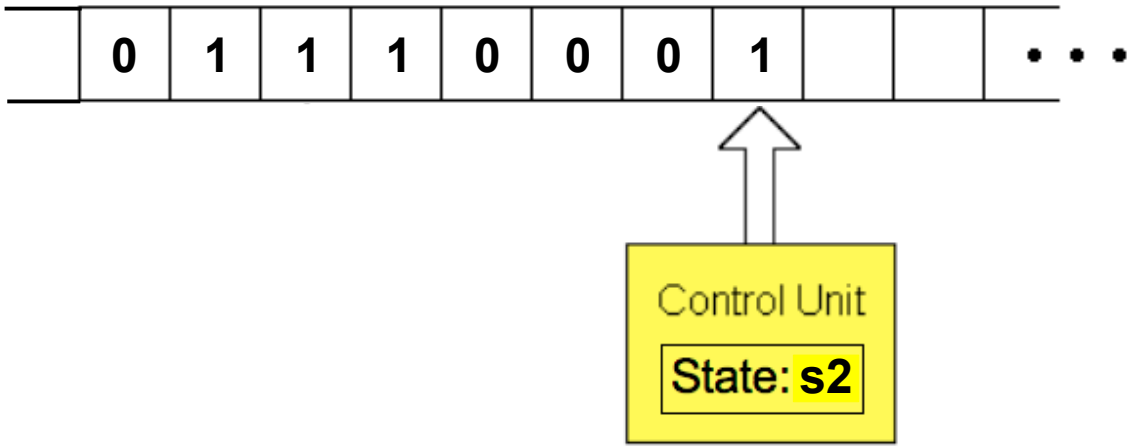
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

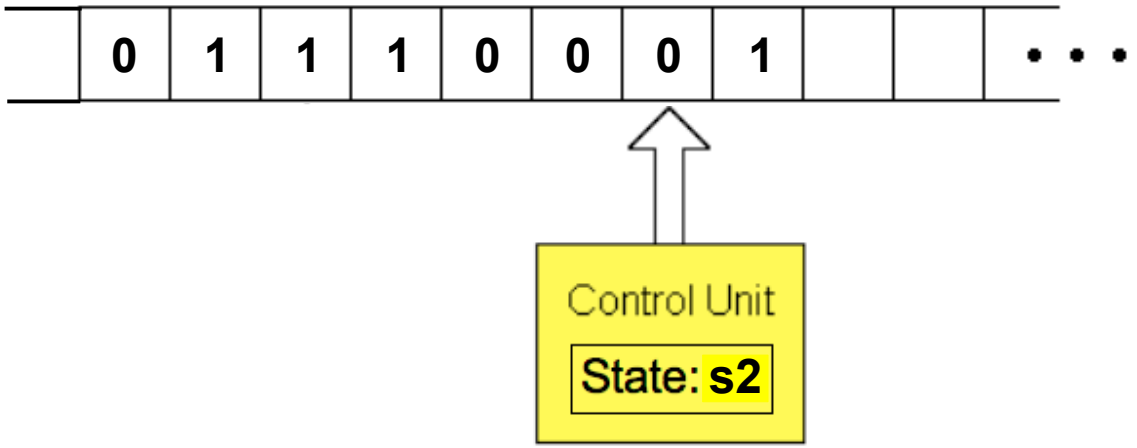
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

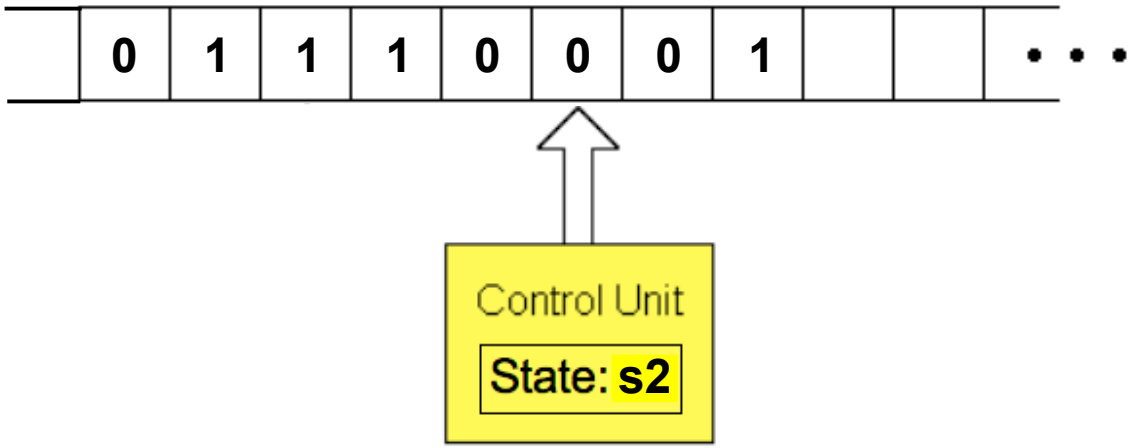
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

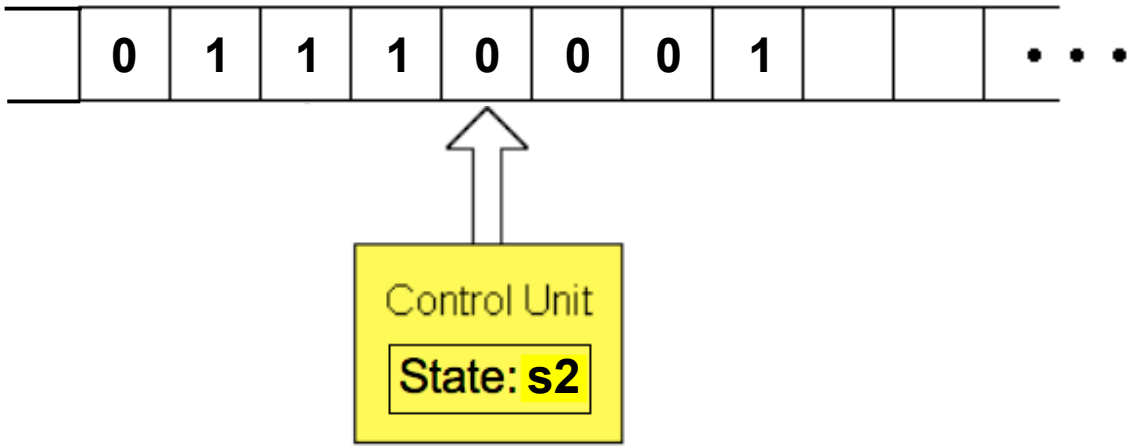
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

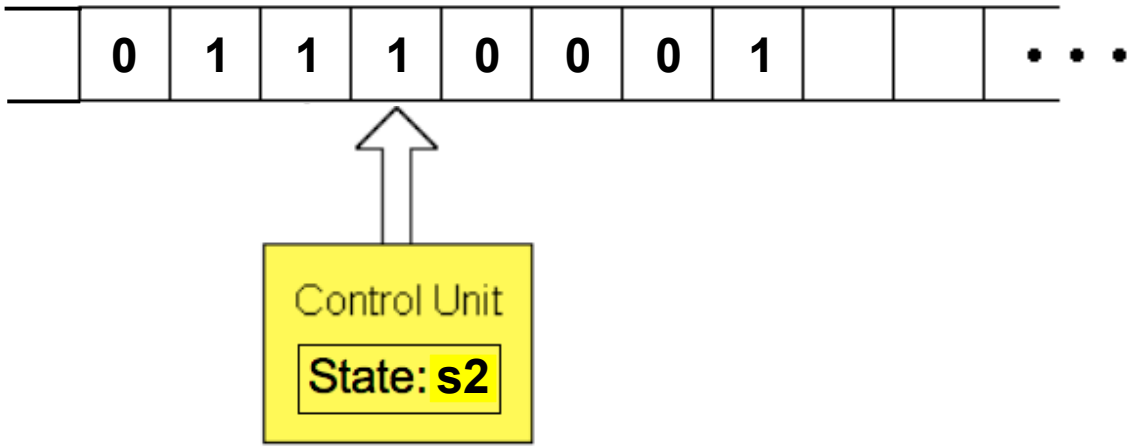
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

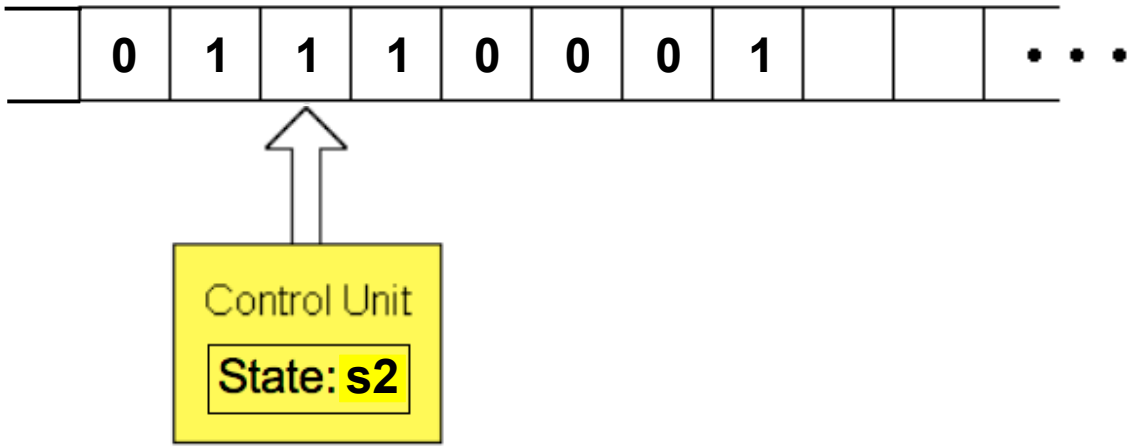
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

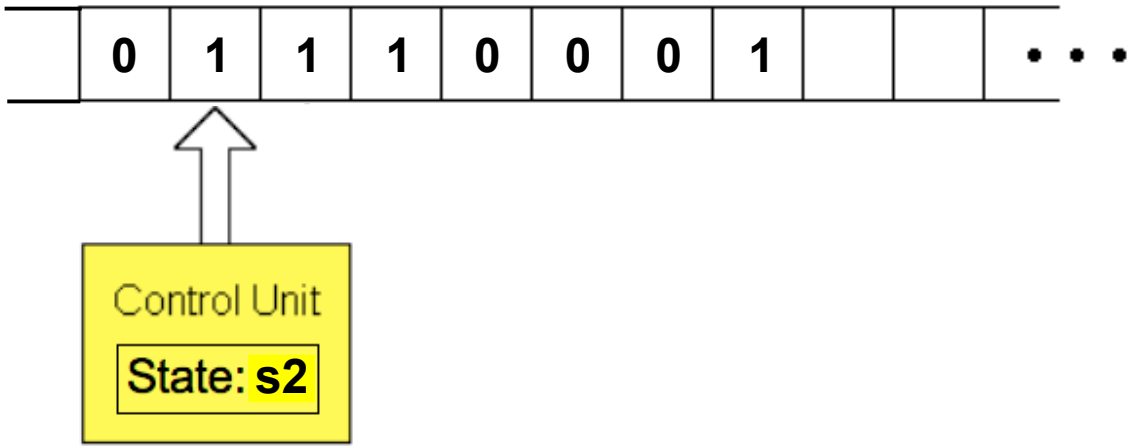
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

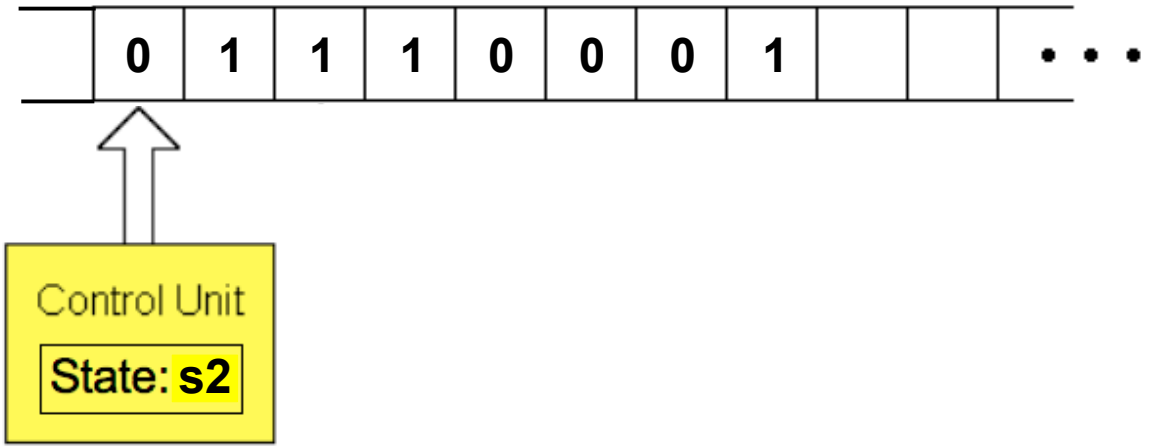
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

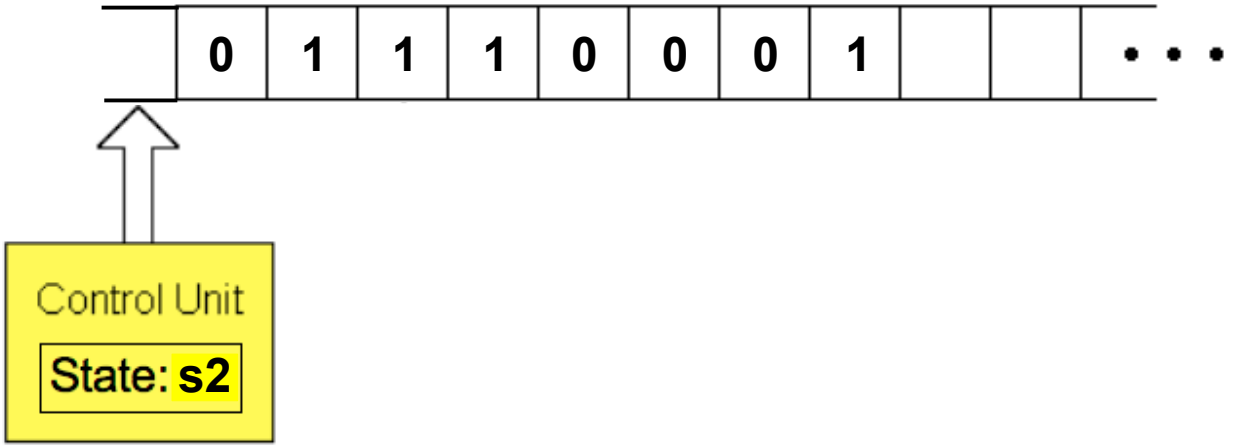
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

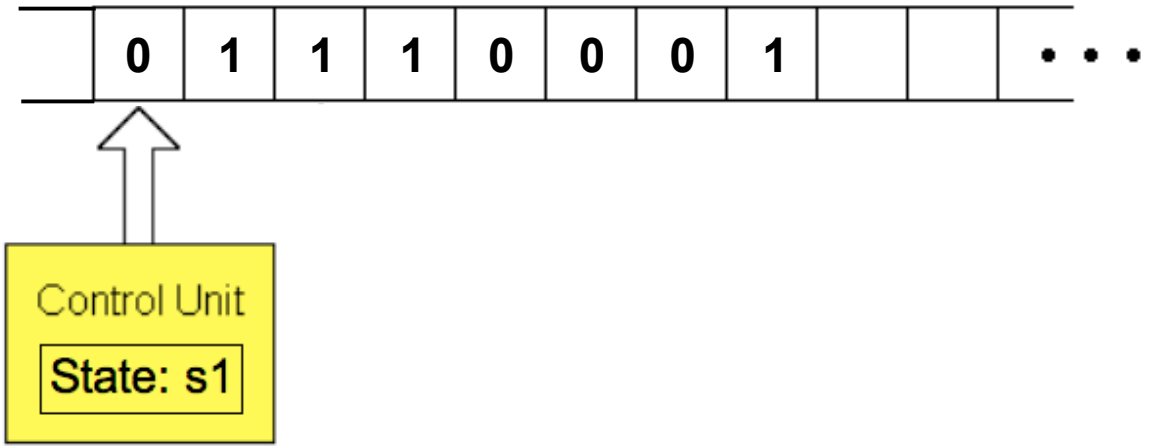
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

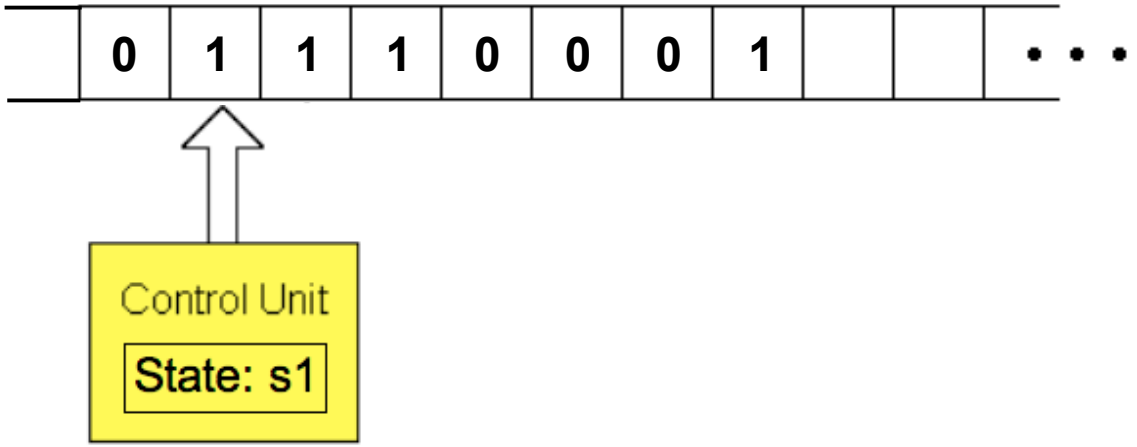
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

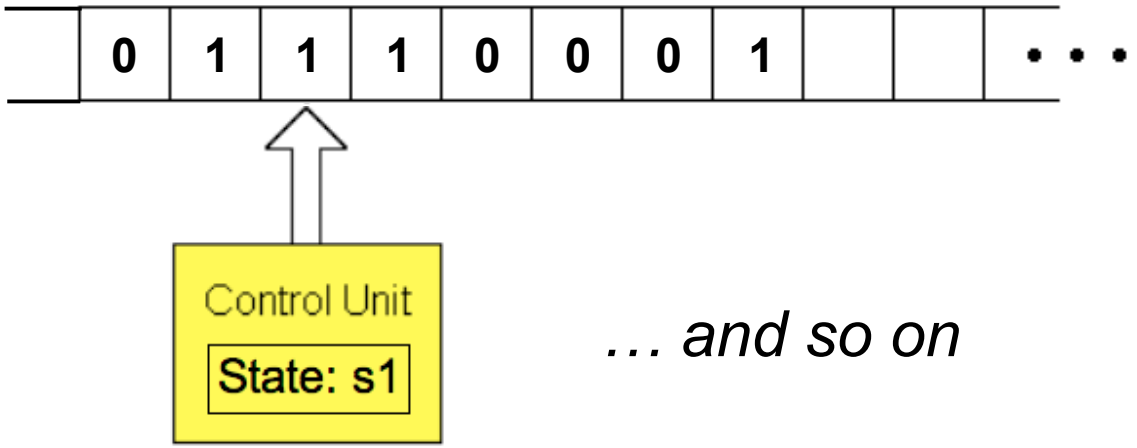
How About a “Zigzag” Machine?



States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**

How About a “Zigzag” Machine?



… and so on

States: **s1**, **s2**

current state	*current symbol*	*new symbol*	*move*	*new state*
s1	**0**	**0**	**right**	**s1**
s1	**1**	**1**	**right**	**s1**
s1	***blank***	***blank***	**left**	**s2**
s2	**0**	**0**	**left**	**s2**
s2	**1**	**1**	**left**	**s2**
s2	***blank***	***blank***	**right**	**s1**