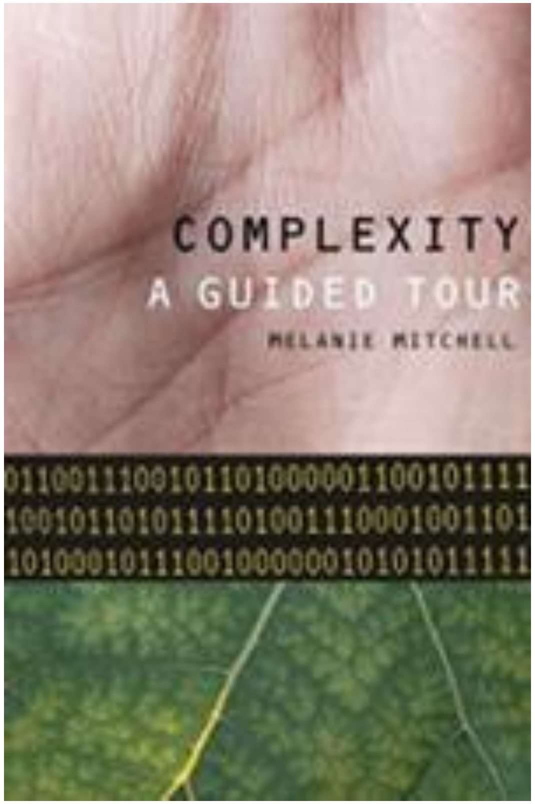
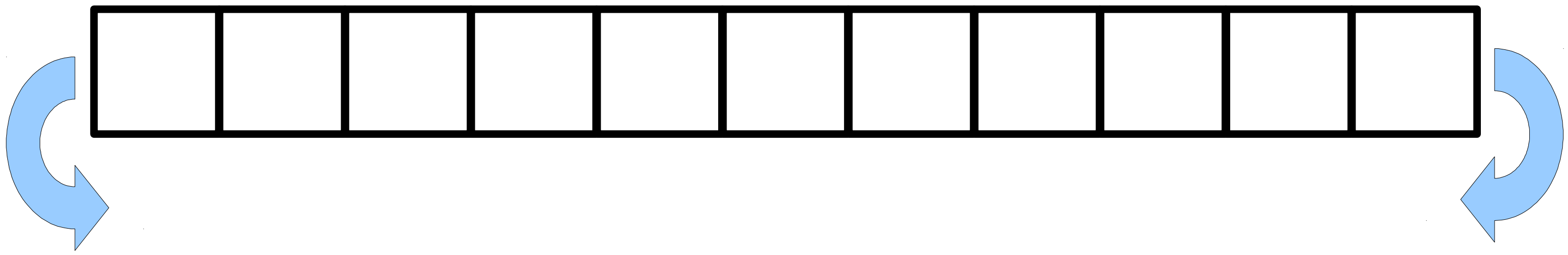
Elementary Cellular Automata

Reading Assignment for Tuesday



Chapter 11 (pages 160-168)

One-Dimensional Cellular Automata



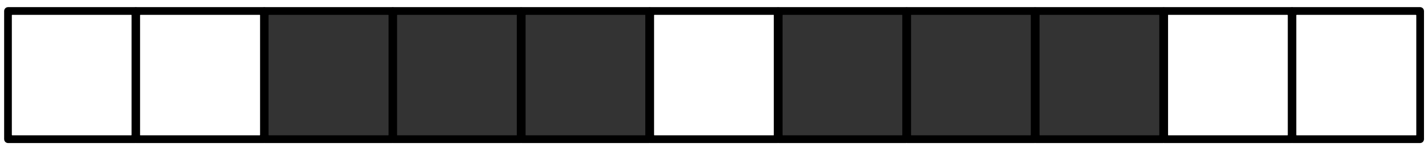
One-Dimensional Cellular Automata

Time step 0

One-Dimensional Cellular Automata

Time step 1

One-Dimensional Cellular Automata

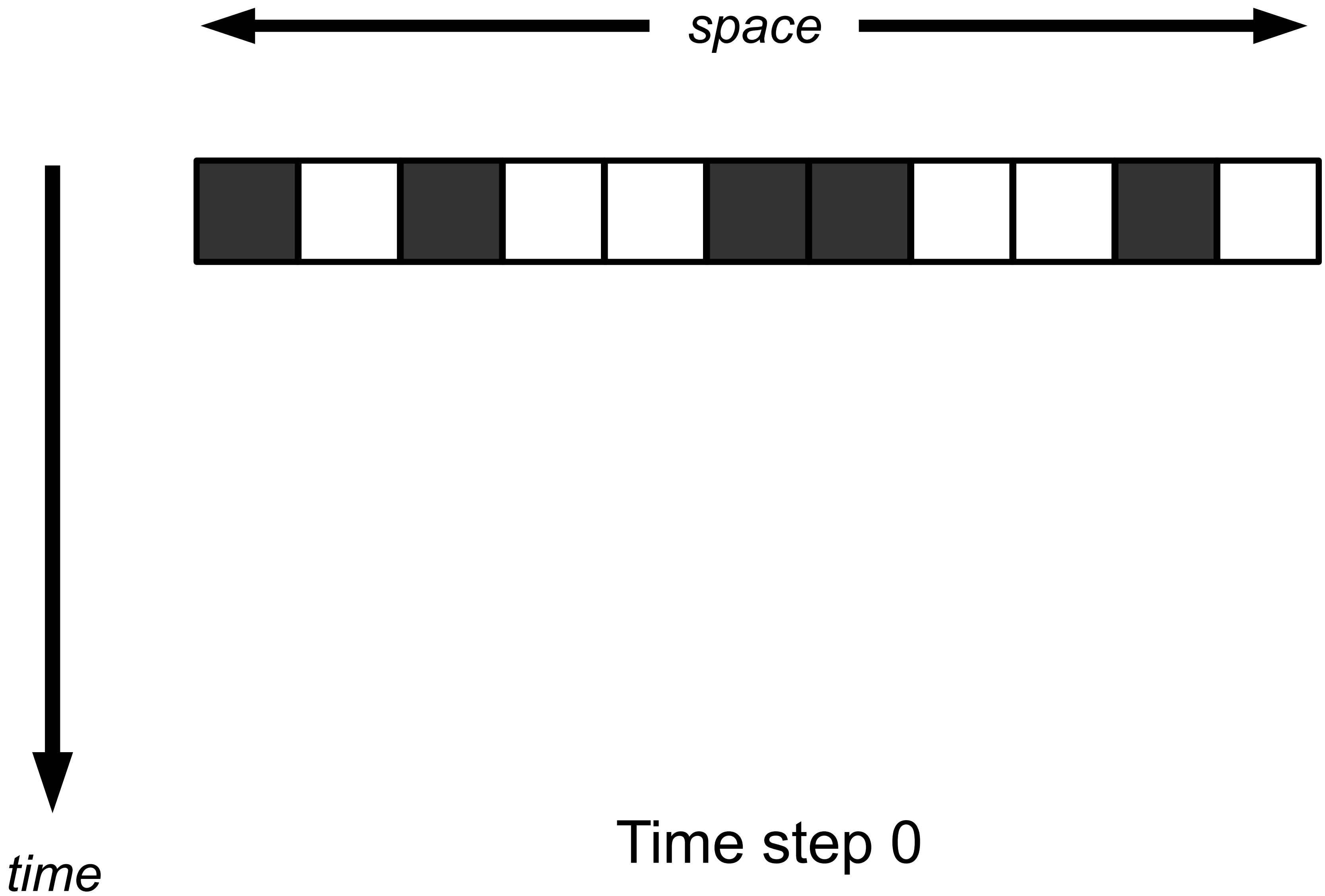


Time step 2

One-Dimensional Cellular Automata

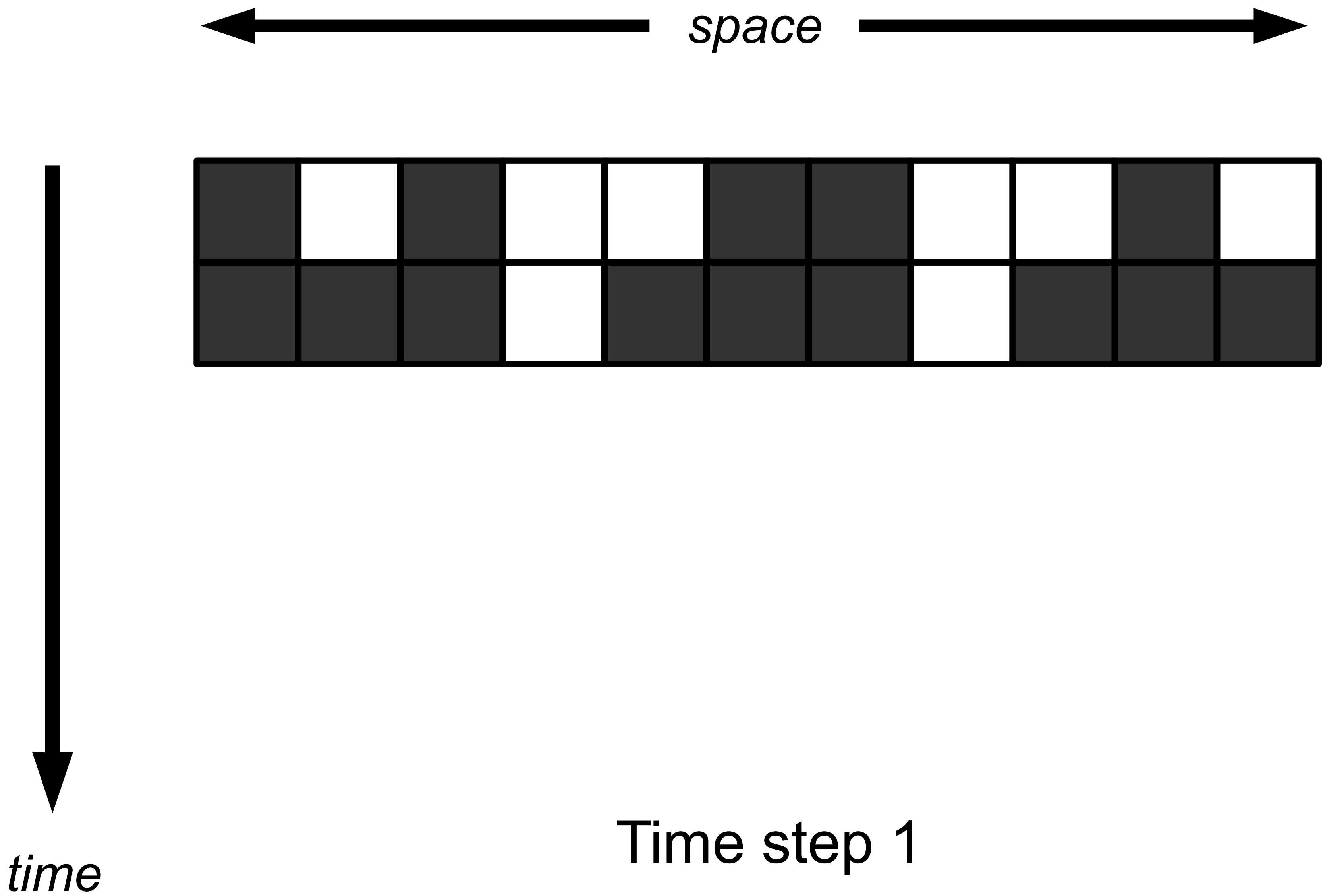
Time step 3

"Space Time" Diagram



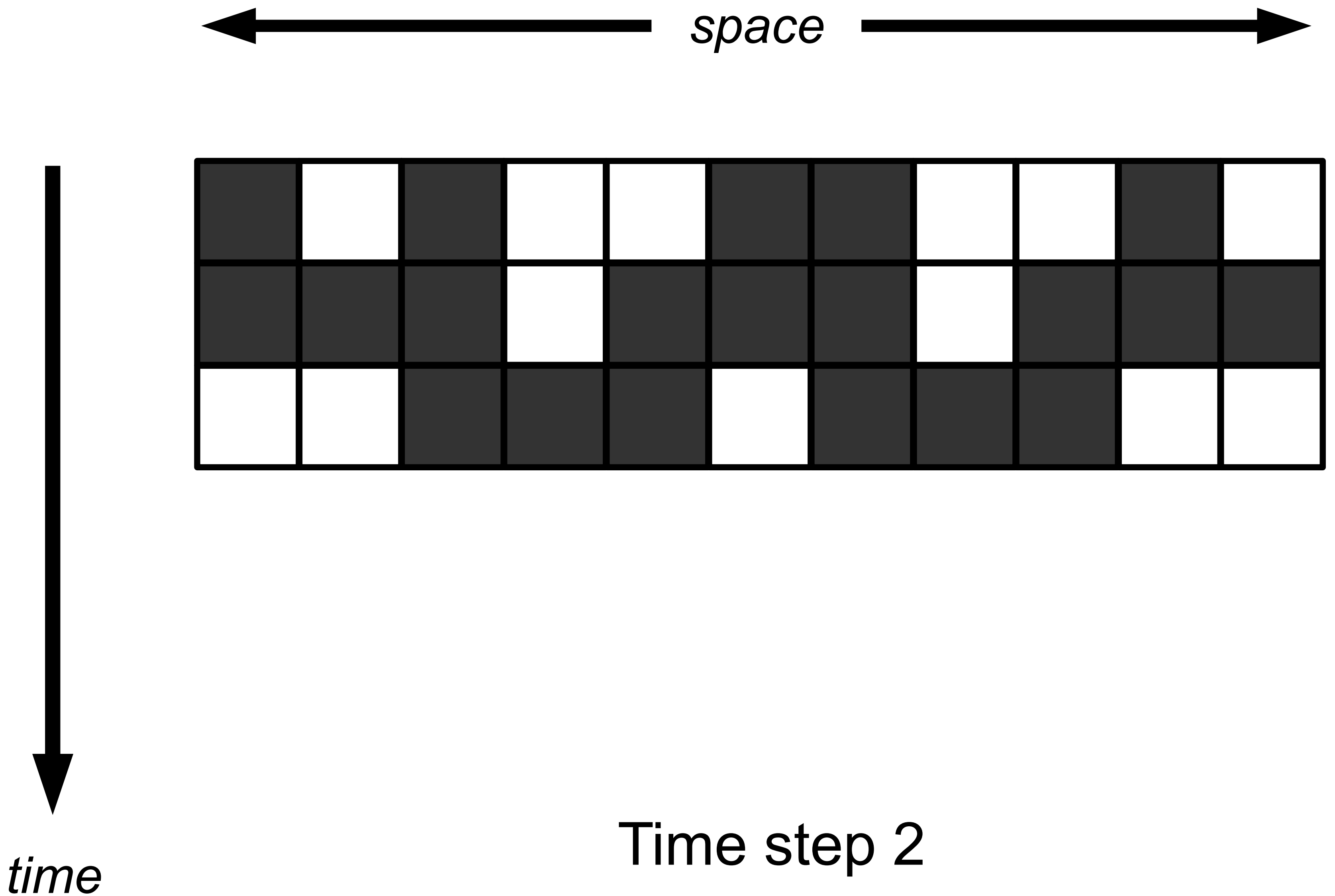
Time step 0

“Space Time” Diagram



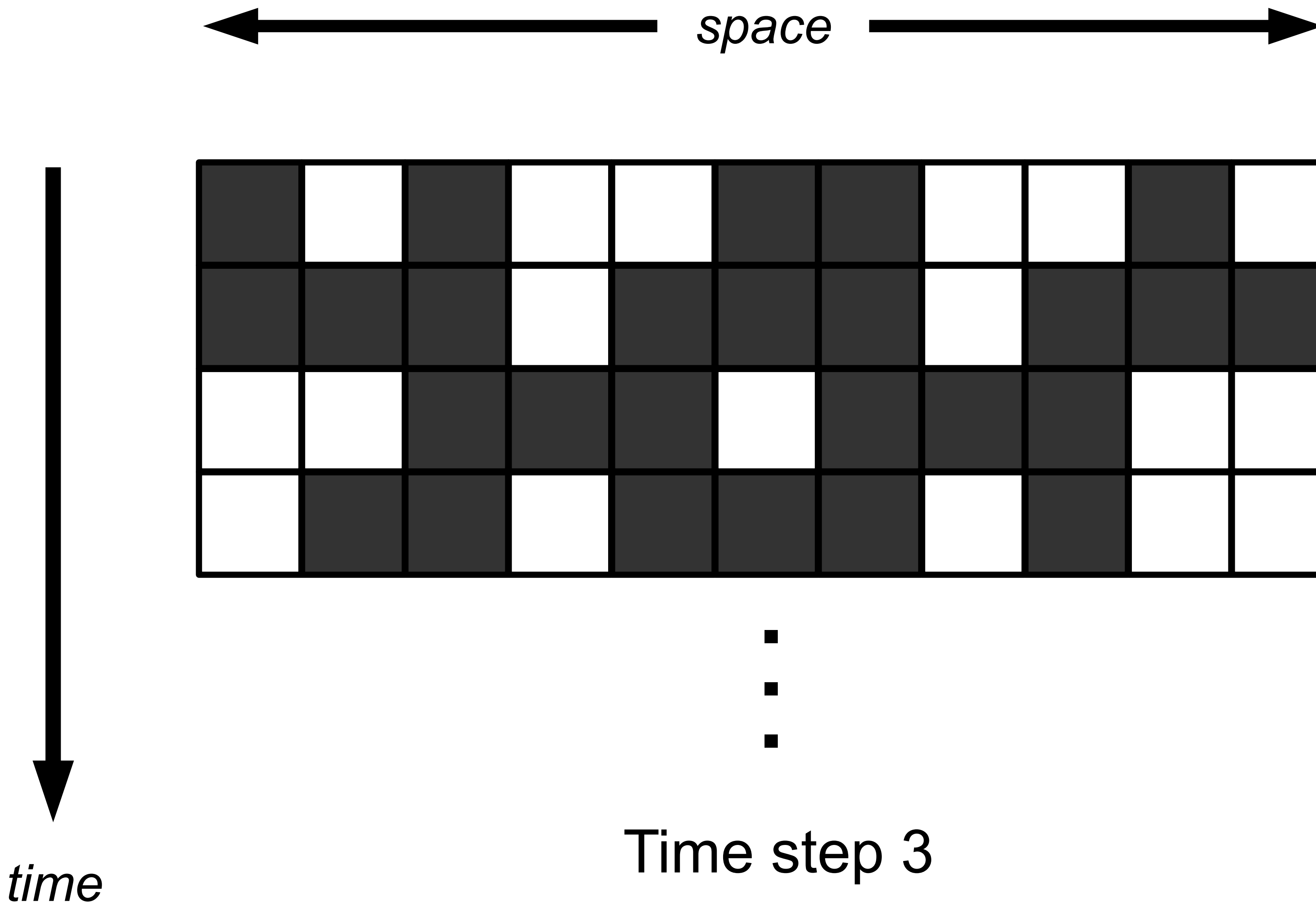
Time step 1

"Space Time" Diagram



Time step 2

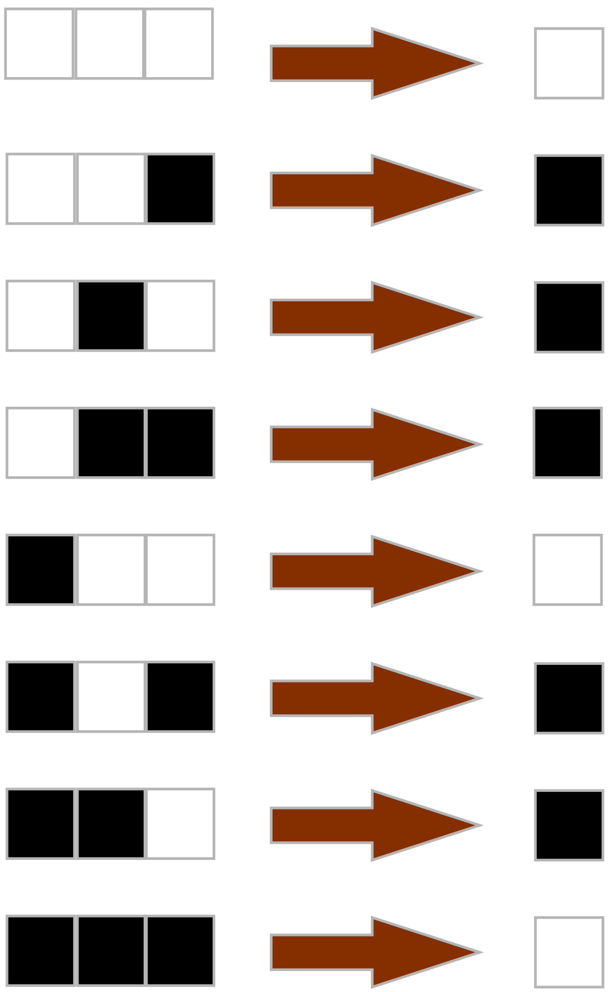
“Space Time” Diagram

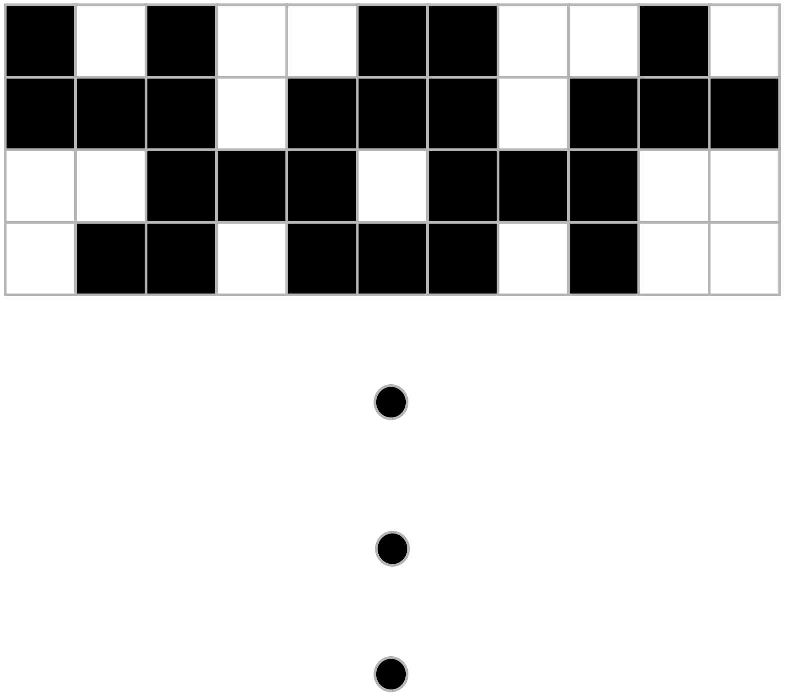


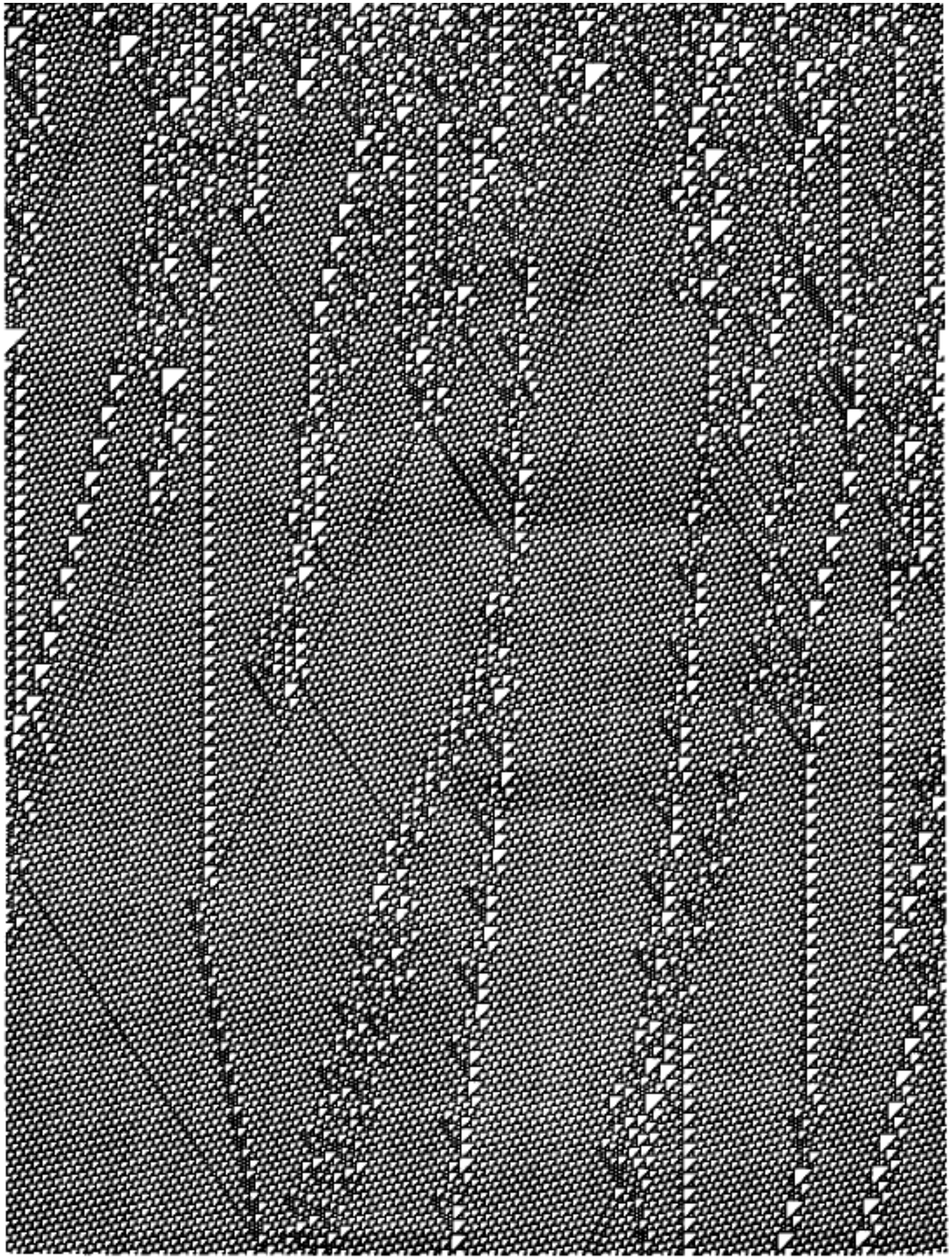
Elementary cellular automata

One-dimensional, two states (black and white)

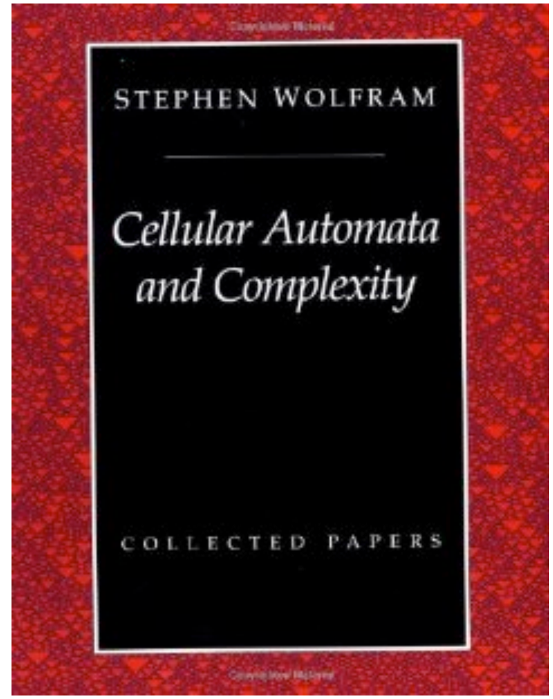
Rule:

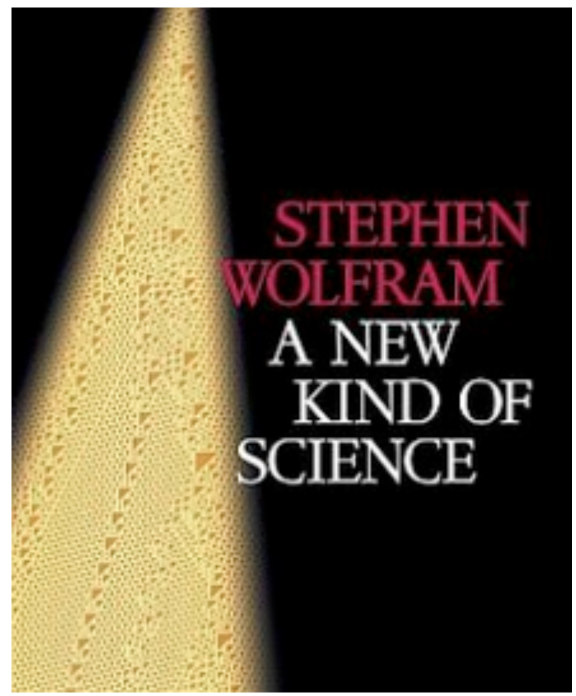






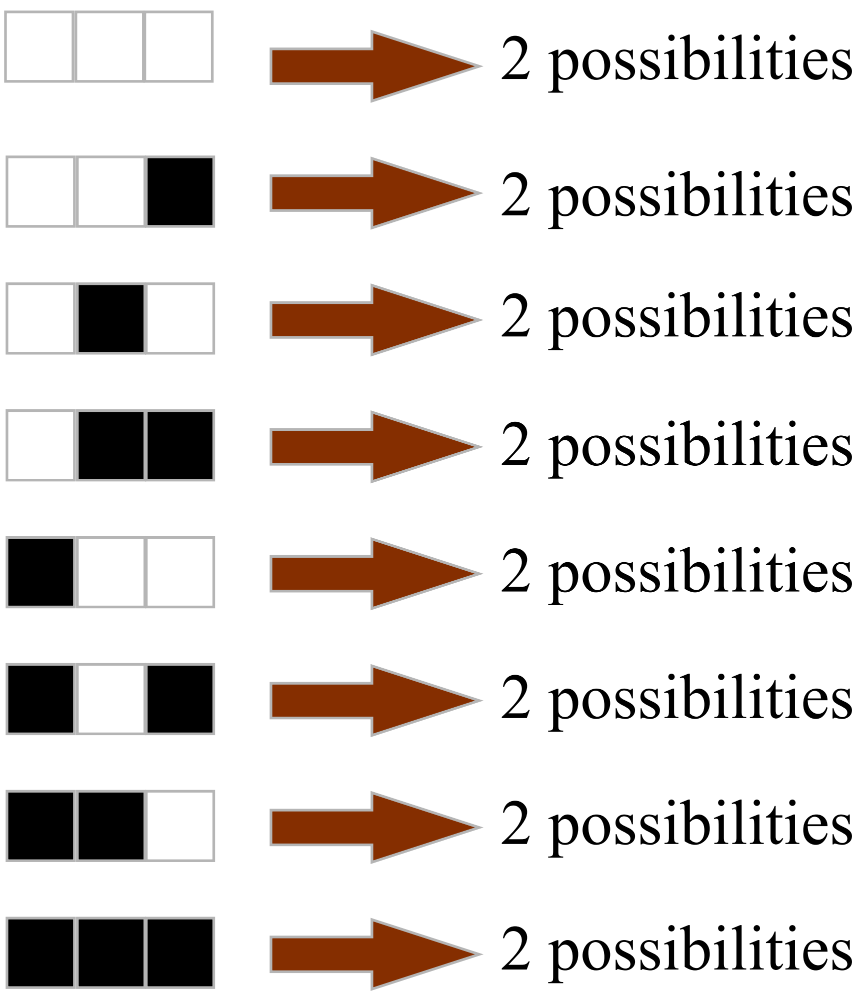
Stephen Wolfram





To define an ECA, fill in right side of arrows with black and white boxes:

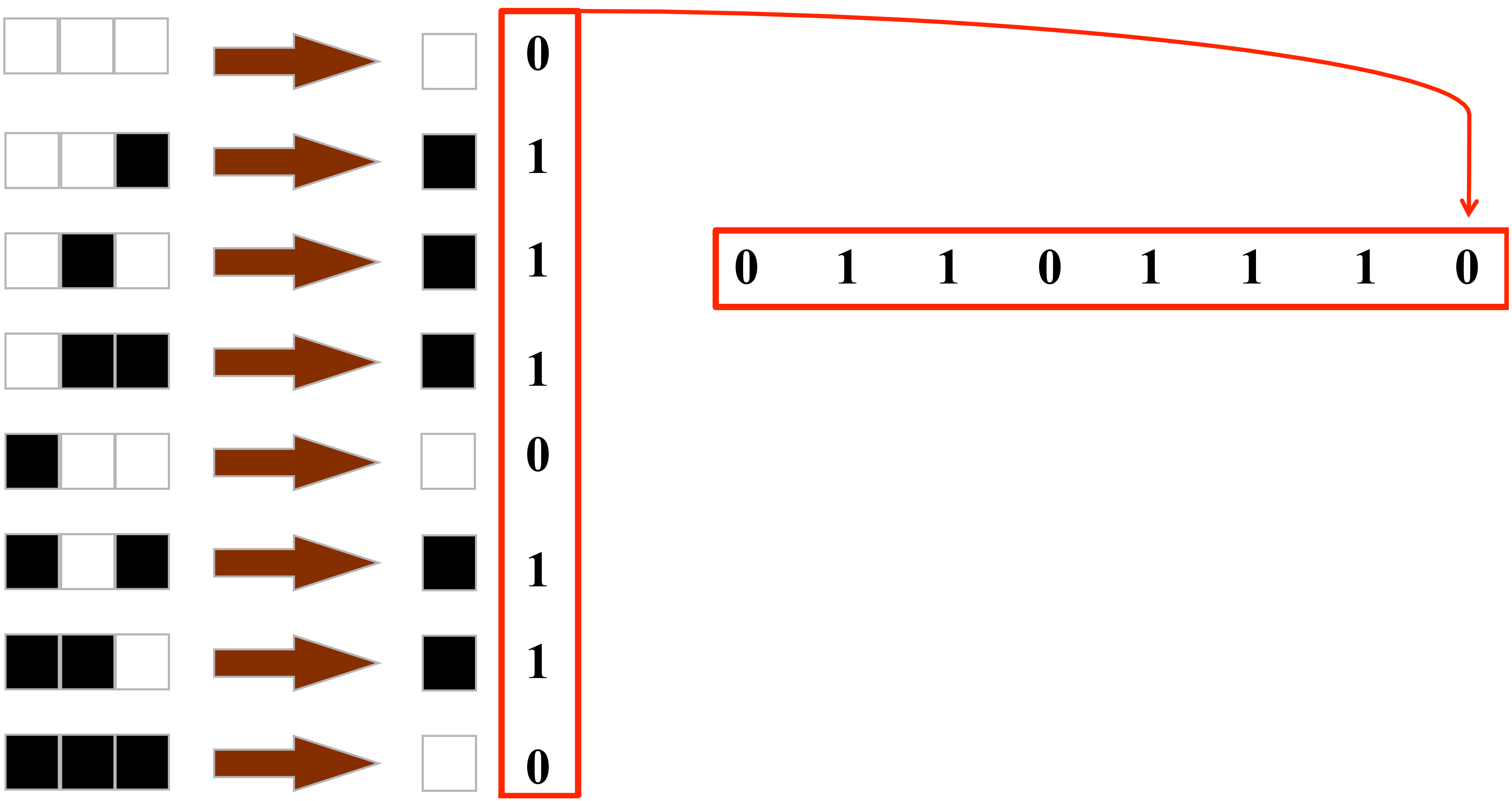
Rule:



Total: 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 = 2^8
= 256 possible ECAs

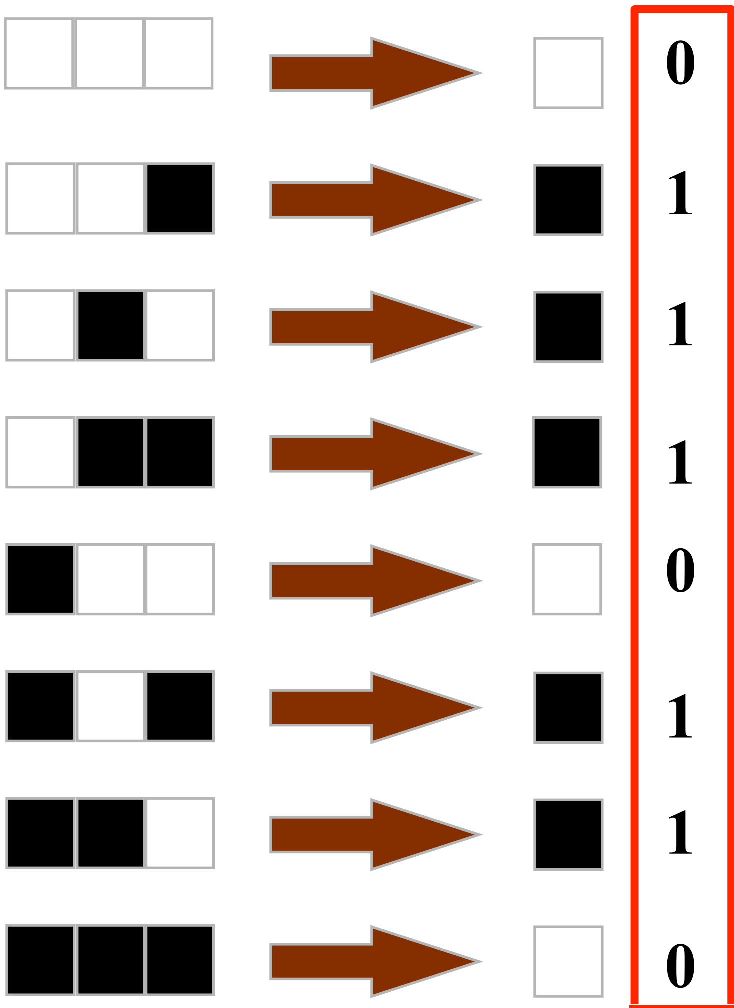
Wolfram numbering:

Rule:



Wolfram numbering:

Rule:



0 1 1 0 1 1 1 0

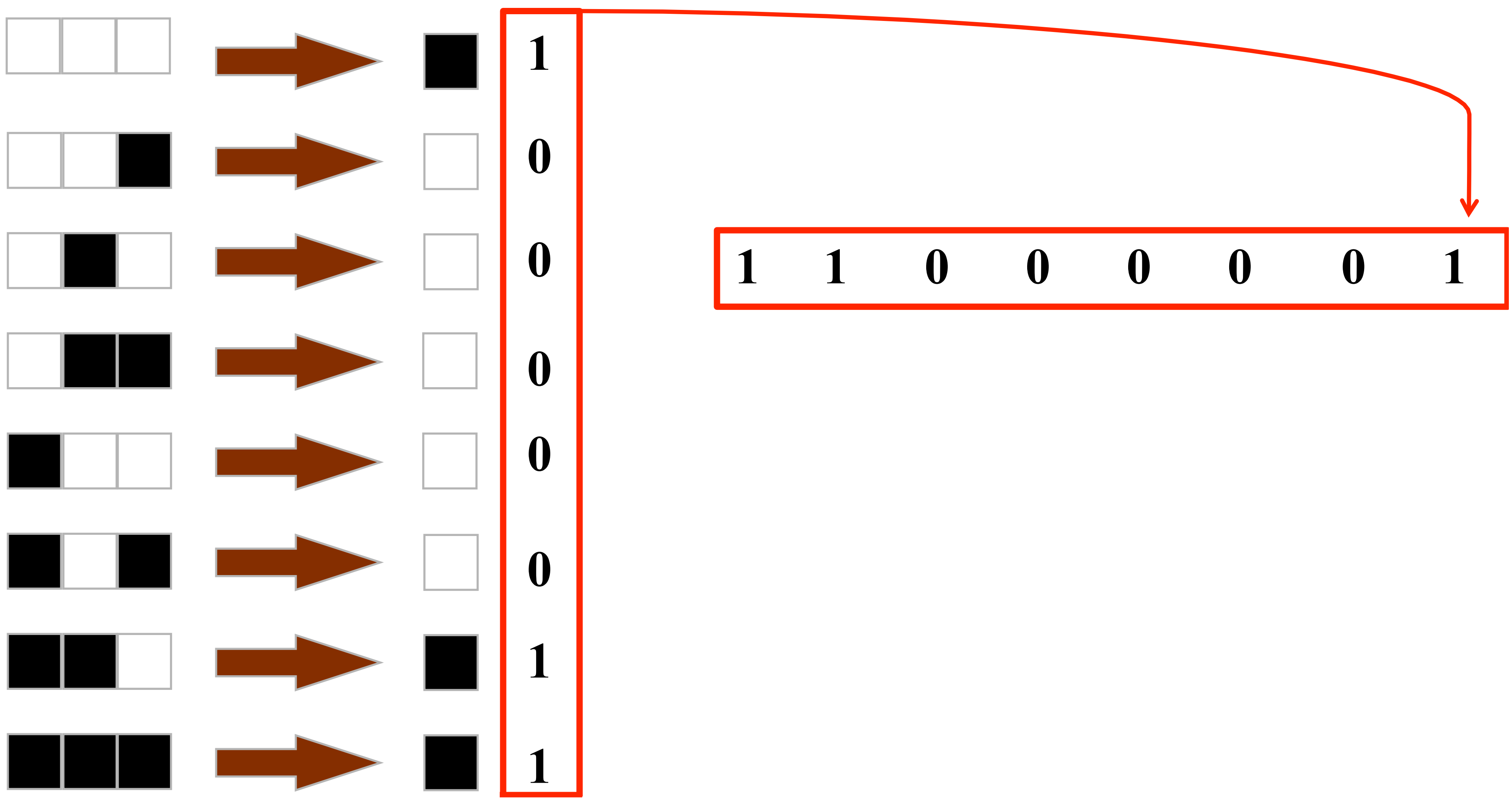
Interpret this as an integer in base 2:

$$(0\times 2^7)+(1\times 2^6)+(1\times 2^5)+(0\times 2^4)$$
$$+(1\times 2^3)+(1\times 2^2)+(1\times 2^1)+(0\times 2^0)$$
$$=110$$

“Rule 110”

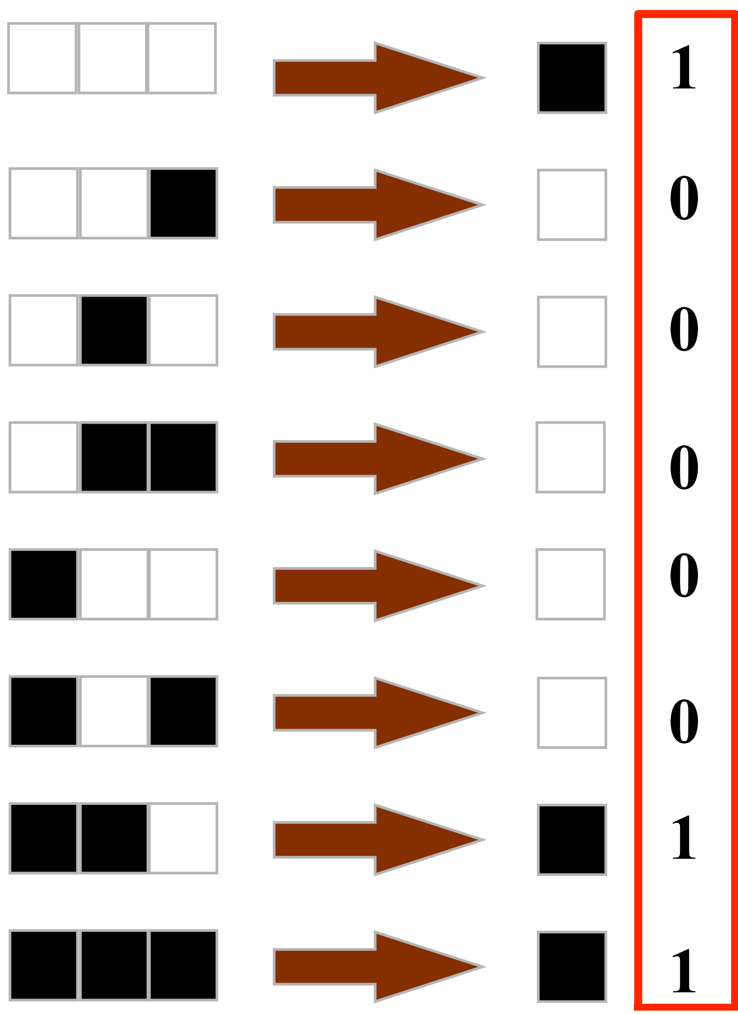
Wolfram numbering:

Rule:



Wolfram numbering:

Rule:



1 1 0 0 0 0 0 1

Interpret this as an integer in base 2:

$$(1\times 2^{7})+(1\times 2^{6})+(0\times 2^{5})+(0\times 2^{4})$$

$$+(0\times 2^{3})+(0\times 2^{2})+(0\times 2^{1})+(1\times 2^{0}$$

$$=128+64+1=193$$

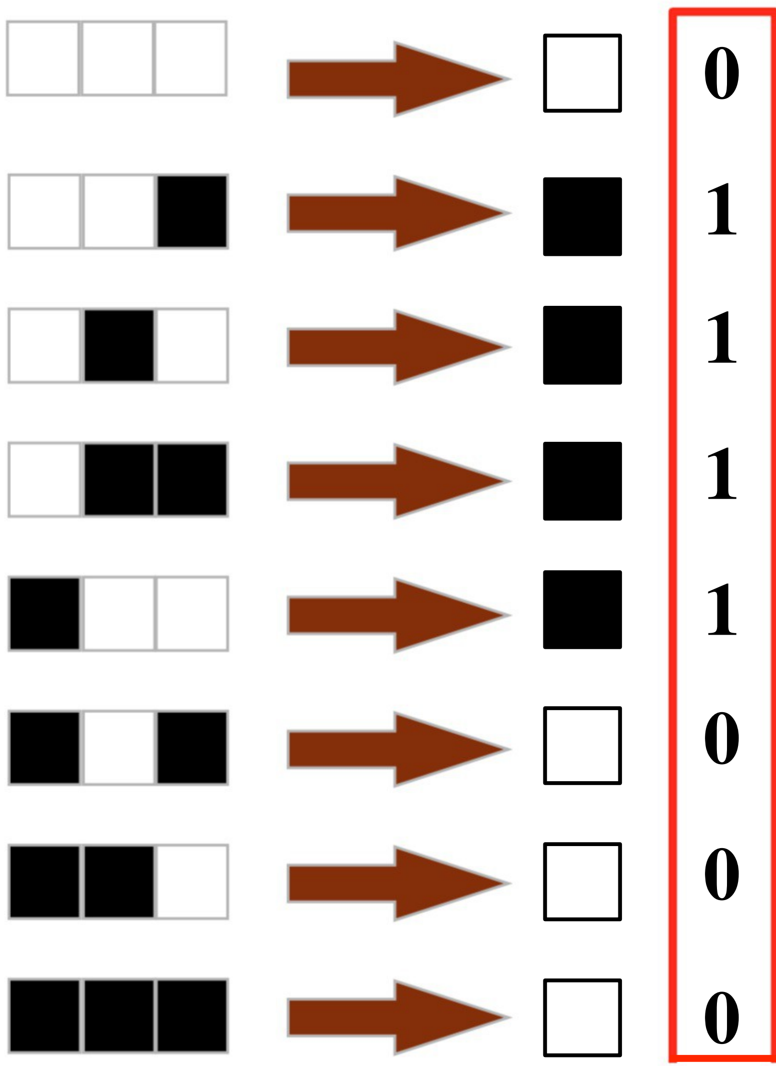
"Rule 193"

"The Rule 30 automaton is the most surprising thing I've ever seen in science....It took me several years to absorb how important this was. But in the end, I realized that this one picture contains the clue to what's perhaps the most long-standing mystery in all of science: where, in the end, the complexity of the natural world comes from."

——Stephen Wolfram (Quoted in *Forbes*)

Wolfram patented Rule 30's use as a pseudo-random number generator!

Rule 30



0 0 0 1 1 1 1 0

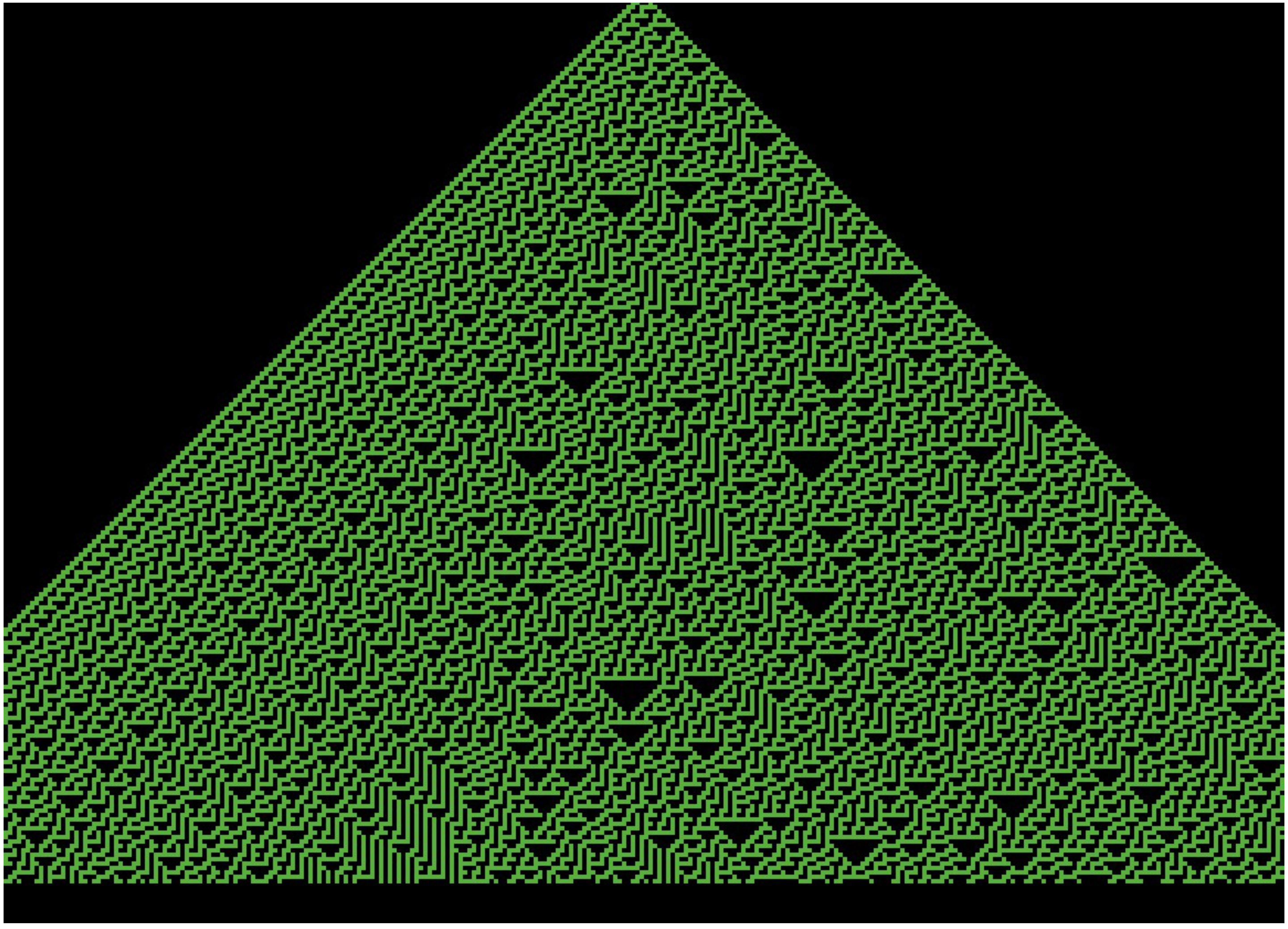
Interpret this as an integer in base 2:

$$(0 \times 2^{7}) + (0 \times 2^{6}) + (0 \times 2^{5}) + (1 \times 2^{4}) + (1 \times 2^{3}) + (1 \times 2^{2}) + (1 \times 2^{1}) + (0 \times 2^{0})$$

$$= 16 + 8 + 4 + 2 = 30$$

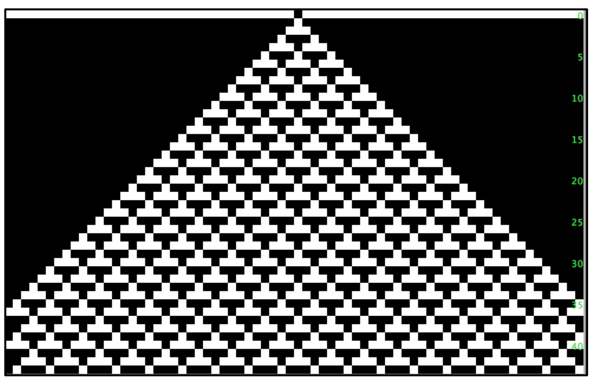
“Rule 30”

NetLogo Demo

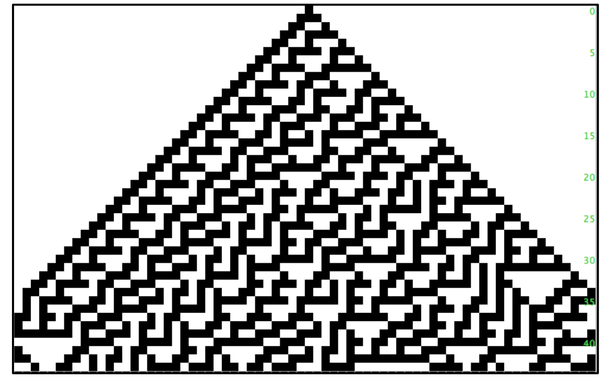


Wolfram's Four Classes of CA Behavior

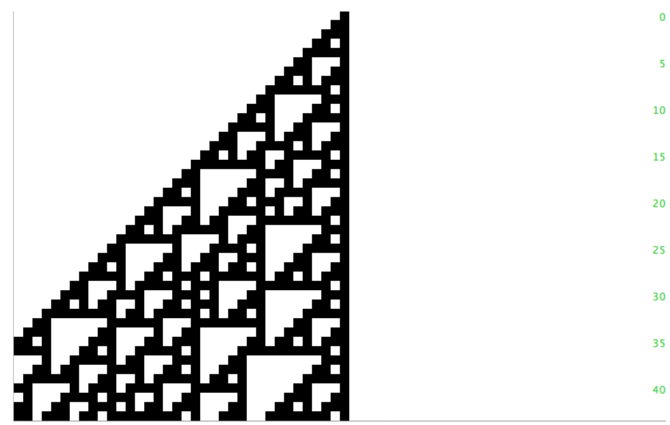
Class 1: Almost all initial configurations relax after a transient period to the same fixed configuration.



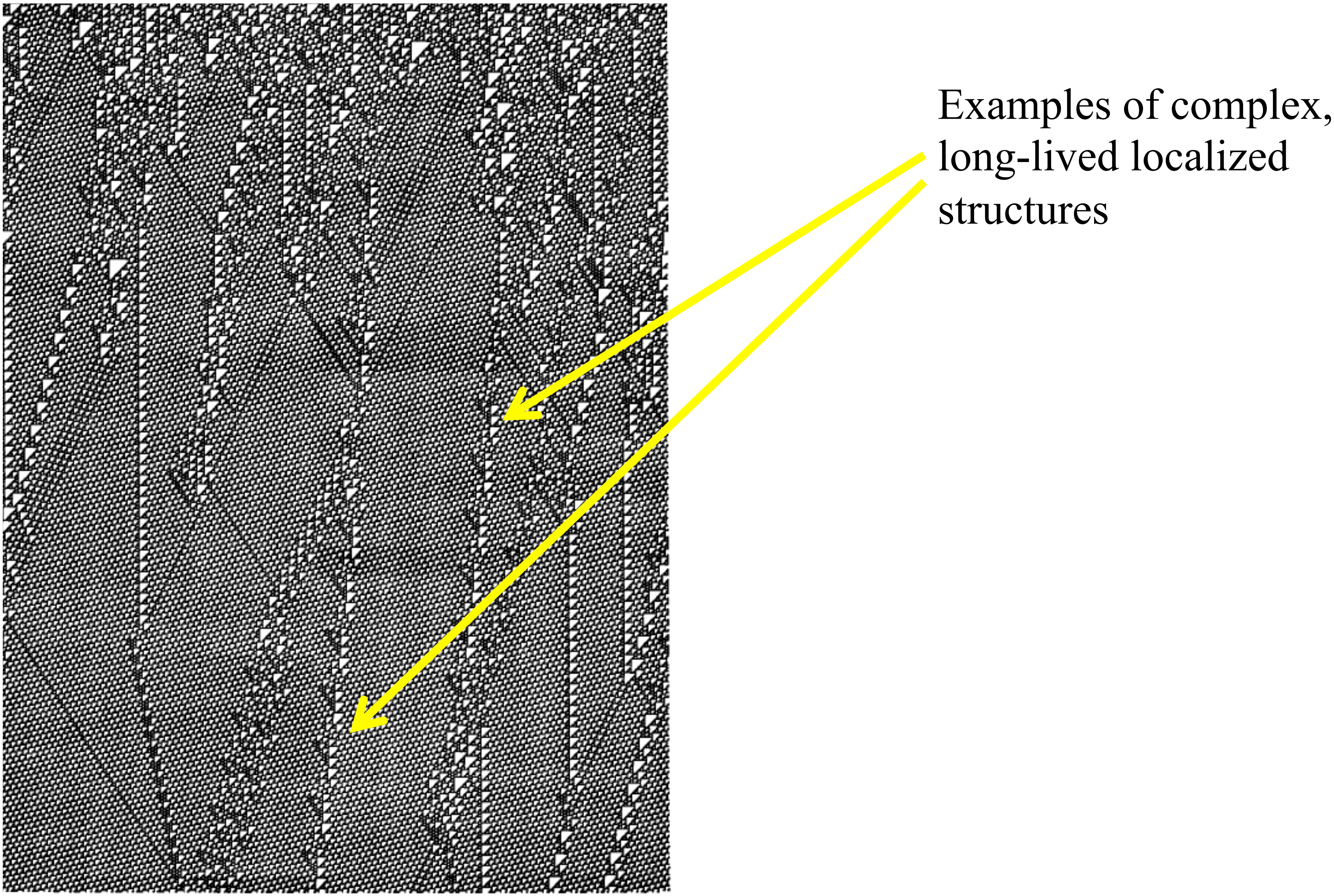
Class 2: Almost all initial configurations relax after a transient period to some fixed point or some periodic cycle of configurations, but which one depends on the initial configuration



Class 3: Almost all initial configurations relax after a transient period to chaotic behavior. (The term ``chaotic'' here refers to apparently unpredictable space-time behavior.)



Class 4: Some initial configurations result in complex localized structures, sometime long-lived.

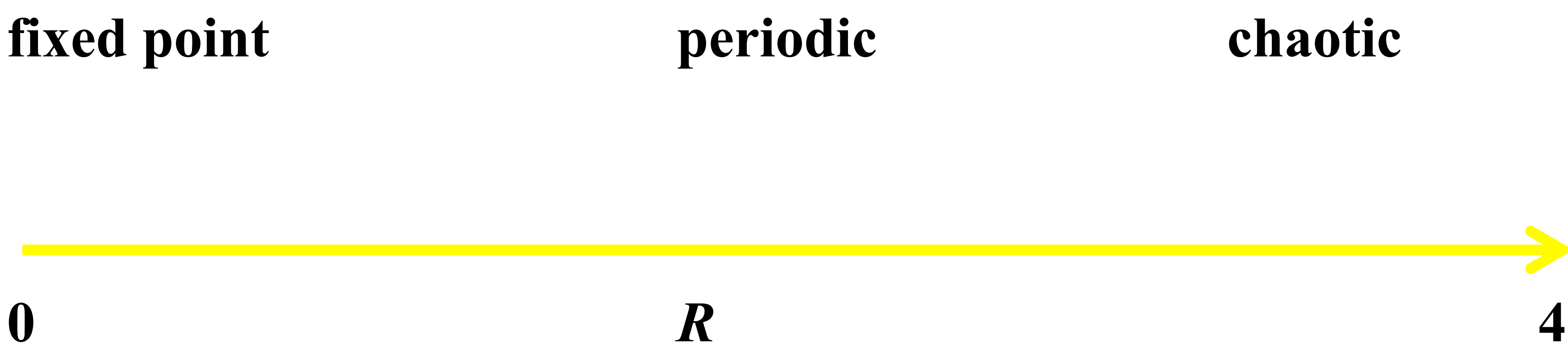


Rule 110

CAs as dynamical systems

(Analogy with logistic map)

Logistic Map	**Elementary Cellular Automata**
$x_{t+1} = f(x_t) = R\,x_t(1 - x_t)$	$lattice_{t+1} = f(lattice_t)$ [f = ECA rule)
Deterministic	Deterministic
Discrete time steps	Discrete time steps
Continuous "state" (value of x is a real number)	Discrete state (value of lattice is sequence of "black" and "white")
Dynamics: Fixed point --- periodic ---- chaos	**Dynamics:** Fixed point – periodic – chaos
Control parameter: R	Control parameter: ?


fixed point
periodic
chaotic
0
R
4

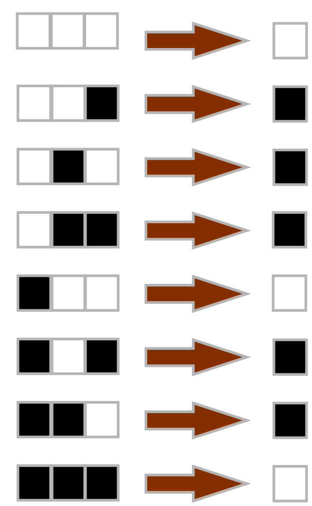
Langton's *Lambda* parameter as a proposed control parameter for CAs

Chris Langton

For two-state (black and white) CAs:

Lambda = fraction of black output states in CA rule table

For example:



Lambda = 5/8

Langton’s hypothesis:

“Typical” CA behavior (after transients):

fixed point periodic chaotic periodic fixed-point

0 **1**

Lambda

(for two-state CAs)

***Lambda* is a better predictor of behavior for neighborhood size > 3 cells**

Summary

- CAs can be viewed as dynamical systems, with different attractors (fixed-point, periodic, chaotic, “edge of chaos”)
- These correspond to Wolfram’s four classes
- Langton’s *Lambda* parameter is one “control parameter” that (roughly) indicates what type of attractor to expect
- The Game of Life is a Class 4 CA!
- Wolfram hypothesized that Class 4 CAs are capable of “universal computation”